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STANDARD ARITHMETIC

BROOKS

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CHRISTOPHER SOWER COMPANY
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THE NORMAL STANDARD ARITHMETIC,

BY ANALYSIS AND INDUCTION,

DESIGNED FOR

PUBLIC AND PRIVATE SCHOOLS, NORMAL
SCHOOLS, ACADEMIES, ETC.

BY

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TRIGONOMETRY, PHILOSOPHY OF ARITHMETIC, METHODS OF
TEACHING, MENTAL SCIENCE AND CULTURE, ETC.

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PREFACE.

THE author's first work on arithmetic was published over thirty years ago. The aim of this work was to unfold the science of numbers with logical simplicity, and to base the art of computation upon an intelligent conception of principles. That the method of treating the subject was correct is attested by the widespread popularity of his works and the influence they have had in shaping correct systems of arithmetical instruction.

The fundamental spirit of the method of treatment was that of analysis. This simple principle, running like a golden thread through the entire treatise, simplified arithmetical conceptions and operations, and bound the subject together into a beautiful and symmetrical unity. It gave the pupil a power of thought and comprehension which enabled him to trace his way through the most complex combinations, and to hold the entire subject clearly in his mental grasp. The union with this analysis of the spirit of induction in deriving generalizations from analytical processes placed in the hands of the student the keys of power and mastery in the science. So potent are these two processes that there arises the maxim,—*Analysis and induction are the golden keys of science.*

While the principles of arithmetic remain unchanged, new inventions and new modes of life introduce new objects to which numbers are applied. The standards of the requirements of instruction vary also with the introduction of new branches of study into the elementary schools. Text-books on arithmetic must therefore be revised every few years to keep pace with these changes and requirements. To meet these demands the present book has been written. In its preparation the aim has been to retain all those methods of teaching the science that gave popularity to the author's former works, and to present such new applications of these methods as may represent the latest forms and requirements of society and business. Especial care has been taken to embody in it the best ideas of modern education, and thus adapt the work to the needs of the modern school-room.

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The two leading objects, therefore, in the preparation of the work are—first, that it should be scientific, and second, that it should be practical. Its scientific character consists in its analytical treatment; its simple and concise definitions; its clear, brief, and logical solutions and explanations; and its philosophical arrangements and classifications. It thus stands opposed to the modern tendency to give pupils a disconnected and fragmentary idea of branches of study, which deprives them of the power that comes from a logical and comprehensive grasp of a subject. The motto of the work has been that the more scientific the treatment the greater its simplicity.

The practical character of the work will be seen by an examination of its problems. The applications of the science are not the thought of the author as to what business may be, but they represent the actual business of the day. The solutions in an arithmetic should be, so far as possible, not merely school-room methods, but the actual methods of the counting-room and business-office. To carry out this idea the author has taken special pains to represent the business of real life. Many of the problems and processes are derived from actual business transactions. The Bills and Accounts came out of the stores; the examples in Percentage, Taxes, Banking, Exchange, etc. represent the real business of the day; and the effort has been made to give a spirit of reality to even those parts of the subject which treat mainly of the principles of numbers.

Many of the solutions and explanations are the product of the author's earlier years as a teacher. His purpose was to give to arithmetical reasoning some of that finish and elegance that belonged to geometry. Many of these solutions were tested with his pupils several years before they were published, so that they are largely the outgrowth of the school-room. Based upon a careful study of the human mind, they are believed to represent the natural processes of the mind in clear and logical thinking upon arithmetical subjects. The aim has been that they shall be simple and direct in thought, so that they may be readily understood by the pupil, and yet so concise and logical that they seem to meet the requirements of a finished demonstration.

Taught to reason in this way, pupils become independent of rules or can derive them for themselves. While rules are presented in the work, they are not designed to be committed or blindly followed. They are merely concise expressions of the methods of operation which may aid the pupils in stating their own processes. They are not given to suggest methods of operation, but merely as concise statements of these operations. In teaching, pupils should be required to see the reason for the different processes, and then to derive their own methods. The rules here given are thus designed as a guide in describing operations, and may sometimes aid the student in remembering them.

The arrangement of the work is believed to be both scientific and practical, being in accordance with the principles of science and adapted to the natural mental growth and development of the pupil. The motto has been from the easy to the difficult, from the simple to the complex, from the known to the unknown. Care has been taken to present the simpler and more practical subjects first, and not to anticipate any principles or processes before the pupil is prepared for them. The proper gradation of the exercises has been kept continually in view. No problem has been inserted, however interesting in its nature, for which it was not thought the pupil had been previously prepared. In the unfolding of subjects each problem was viewed as a stepping-stone to those which followed it, and thus problems which might seem difficult in themselves become easy by virtue of the pupil's previous preparation for them. The proper grading of the path of the science of numbers, with here and there a flower of interest planted along its border, has been a prominent object in the mind of the author.

In the scope and extent of the work judgment has been exercised to present the average requirements of elementary schools. Several subjects heretofore presented in arithmetic have been omitted, while others have been given at the close of the work under the head of a Supplement. The work is regarded as presenting a complete course for elementary schools without the Supplement; but teachers who desire a fuller course will find several topics in the Supplement which are usually found in text-books of this grade. The work is thus adapted to meet the demands of all grades of elementary schools.

With a grateful sense of the appreciation extended to my previous works on arithmetic, I offer the present treatise on the science to teachers and others having charge of the education of youth, with the sincere hope that it may be found a valuable aid in imparting a knowledge of the science of numbers and in developing the thought-powers of the youth of the country.

EDWARD BROOKS.

PHILADELPHIA, *May* 10, 1895.

SECTION VIII.

PERCENTAGE.

Simple Percentage 238
Trade Discounts 248
Profit and Loss 249
Commission 255
Stocks and Dividends . . . 259
Premium and Discount . . 263
Brokerage 266
Income from Investments . 270
General Taxes 273
Duties 276
Property Insurance 278
Life Insurance 281
Simple Interest 282
Promissory Notes 289
Partial Payments 293
Problems in Interest . . . 297
Discount and Present Worth 301
Bank Discount 302
Exchange 305

SECTION IX.

RATIO AND PROPORTION.

Ratio 312
Simple Proportion 314
Compound Proportion . . . 319
Partnership,
Simple 321
Compound 322
Equation of Payments 324

SECTION X.

INVOLUTION AND EVOLUTION.

Involution 328
Squaring Numbers . . . 330
Cubing Numbers 331
Evolution 333
Square Root 336
Right Triangles 339
Similar Figures 340
Cube Root 342
Similar Volumes 345
Higher Roots 347

SECTION XI.

MENSURATION.

Mensuration of Surfaces . 348
The Triangle 349
The Quadrilateral . . . 349
The Circle 351
The Ellipse 353
Mensuration of Volumes . 354
The Prism 354
The Pyramid 355
The Cylinder 356
The Cone 356
The Frustum 357
The Sphere 358
Gauging 359

MISCELLANEOUS EXAMPLES.

Fundamental Rules 361
Common Fractions 363
Decimal Fractions 365
Bills and Accounts 367
Denominate Numbers 367
Practical Measurements 370
Percentage 373
Interest and Discount 378
Ratio and Proportion 379
Involution and Evolution . . . 382
Mensuration 384

SUPPLEMENT.

Divisor and Multiple 387
Interest by Cancellation . . . 387
Annual Interest 388
Compound Interest 389
Averaging Accounts 390
Settlement of Accounts 392
Geometrical Evolution 393
Arithmetical Progression . . . 396
Geometrical Progression . . . 399
Infinite Series 401
Metric System 402
Notes on Weights and Measures 408
Lumbermen's Notation . .

INTRODUCTION TO PERCENTAGE.

ORAL EXERCISES.

1. A gain of $4 on $10 is a gain of how many dollars on *the hundred?*

SOLUTION.—If the gain on $10 is $4, on $100, which is 10 times $10, the gain is 10 times $4, which is $40.

2. A gain of $2 on $5 is a gain of how many dollars on *the hundred?*

3. What is the gain on a hundred when the gain is 3 on 20? 4 on 20? 5 on 25?

4. If the gain on $100 is $25, what is the gain on $4? On $12? On $20? On $24? On $5? On $15? On $30? On $40?

5. If the gain on $100 is $20, what is the gain on $1? On $12? On $24? On $36?

6. If the gain on $100 is $20, what part of the $100 equals the gain?

7. If the gain on $100 is $40, what part of the $100 equals the gain?

8. What is the gain on $48 at the *rate* of 25 on the 100?

9. What is the gain on $35 at the *rate* of 20 on the 100?

10. What is the gain on $40 at the *rate* of 10 on the *hundred?*

11. What is the gain on $200 at the *rate* of 20 on the *hundred?*

12. What is the gain on $360 at the *rate* of 15 on the *hundred?*

13. What is the *rate per hundred* at a gain of $8 on $40?

14. What is the *rate per hundred* at a gain of $15 on $60?

15. *Per cent.* means the same as *per hundred;* what, then, can we call the *rate per hundred?* *Ans. Rate per cent.*

16. A gain of $15 on $60 is a gain of what *per cent.?*

17. A loss of $12 on $60 is a loss of what *per cent.?*

18. What *per cent.* is a gain of 30 on 60? 4 on 20? 3 on 60? 4 on 80? 8 on 200?

19. What is 10 per cent. of 40? 25 per cent. of 24? 20 per cent. of 40? 50 per cent. of 48?

SOLUTION.—10 per cent. is at the rate of 10 on the 100, and since 10 is $\frac{1}{10}$ of 100, 10 per cent. of 40 is $\frac{1}{10}$ of 40, which is 4.

20. What is 50 per cent. of 28? 30 per cent. of 40? 60 per cent. of 35? 40 per cent. of 45? 20 per cent. of 60?

21. What shall we call the operations which treat of rate per cent.? *Ans. Percentage.*

SECTION VIII.

PERCENTAGE.

341. **Percentage** is the process of computation in which the basis of comparison is a *hundred.*

342. The **Term** *per cent.*—from *per*, by, and *centum*, a hundred—means *by* or *on the hundred.* Thus, 6 per cent. of any quantity means 6 of every hundred of the quantity.

343. The **Symbol** of percentage is %. The per cent. may also be indicated by a common fraction or a decimal. Thus, $6\% = \frac{6}{100} = .06$; $25\% = \frac{25}{100} = .25$.

344. The **Quantities** considered are the *Base*, the *Rate*, the *Percentage*, and the *Amount* or *Difference.*

345. The **Base** is the number on which the percentage is computed.

346. The **Rate** is the number of hundredths of the base which are taken.

347. The **Percentage** is the number which is a certain per cent. of the base.

348. The **Amount** is the sum of the base and percentage. The *Difference* is the difference of the base and percentage.

349. The *Amount* and *Difference* may both be embraced under the general term *Proceeds.*

In computation the rate is usually expressed as a decimal.

EXPRESSION OF THE RATE.

1. Express 5% as a decimal and common fraction.

SOLUTION.—Since 5 per cent. is 5 on a *hundred*, 5% of a quantity is .05 of it; or, as a *fraction*, $\frac{5}{100}$ or $\frac{1}{20}$ of it.

OPERATION.

$$5\% = .05 = \frac{5}{100} = \frac{1}{20}$$

Express as a decimal and common fraction:

2. 4%.
3. 6%.
4. 7%.
5. 8%.
6. 10%.
7. 20%.
8. 25%.
9. 50%.
10. 75%.
11. $6\frac{1}{4}\%$.
12. $8\frac{1}{3}\%$.
13. $11\frac{1}{9}\%$.
14. $12\frac{1}{2}\%$.
15. $16\frac{2}{3}\%$.
16. $33\frac{1}{3}\%$.
17. $37\frac{1}{2}\%$.
18. $62\frac{1}{2}\%$.
19. $83\frac{1}{3}\%$.
20. $87\frac{1}{2}\%$.
21. $\frac{1}{4}\%$.
22. $\frac{1}{2}\%$.
23. $\frac{3}{4}\%$.
24. $\frac{5}{8}\%$.
25. $\frac{15}{16}\%$.

Express in per cent. decimally:

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{5}$.
4. $\frac{1}{10}$.
5. $\frac{3}{4}$.
6. $\frac{1}{8}$.
7. $\frac{1}{9}$.
8. $\frac{1}{12}$.
9. $\frac{1}{3}$.
10. $\frac{1}{6}$.
11. $\frac{3}{8}$.
12. $\frac{5}{8}$.
13. $\frac{1}{25}$.
14. $\frac{1}{50}$.
15. $\frac{1}{16}$.
16. $\frac{37}{40}$.

350. Cases.—The subject of percentage is conveniently treated under three distinct cases:

1. Given the base and the rate, to find the percentage or proceeds.

2. Given the percentage or proceeds and the rate, to find the base.

3. Given the percentage or proceeds and the base, to find the rate.

CASE I.

351. Given the base and the rate, to find the percentage or the proceeds.

ORAL EXERCISES.

1. What is 20% of 150 yards?

SOLUTION.—20% of anything is $\frac{20}{100}$, or $\frac{1}{5}$ of it, and $\frac{1}{5}$ of 150 yards is 30 yards. Therefore, etc.

What is

2. 20% of 65 oz.?
3. 25% of 72 bu.?
4. 50% of 112 mi.?
5. 75% of 108 tons?
6. 15% of 80 yd.?
7. 45% of 140 qt.?
8. $12\frac{1}{2}\%$ of 128 cows?
9. $16\frac{2}{3}\%$ of 120 horses?

10. A grocer bought 120 dozen oranges, and 25% turned out to be bad; how many did he lose?

11. From a hogshead of wine, containing 108 gallons, $33\frac{1}{3}$% was drawn out; how many gallons remained?

12. A train of cars running 40 miles an hour slackens its speed 45%; what is the rate of running after the decrease?

13. A salesman received \$36 a month, but at the beginning of the year his salary was raised $11\frac{1}{9}$%; what did he then receive a month?

14. In the 10th Problem, which is the base, which the rate, and which the percentage?

WRITTEN EXERCISES.

1. What is 5% of \$246? What is the amount of \$246, increased by 5% of itself?

SOLUTION.—5% of \$246 equals .05 times \$246, which, by multiplying, we find to be \$12.30.

OPERATION.

$$\begin{array}{r} \$246 \\ .05 \\ \hline \$12.30 \end{array}$$

SOLUTION.—A number increased by 5%, or .05 times itself, equals 1.05 times itself; 1.05 times \$246 equals \$258.30.

OPERATION.

$$\begin{array}{r} \$246 \\ 1.05 \\ \hline \$258.30 \end{array}$$

Rule.—I. *To find the percentage, multiply the base by the rate.*

II. *To find the proceeds, multiply the base by* 1 *plus the rate, or by* 1 *minus the rate.*

1. When the rate gives a small common fraction, take such a part of the base as is indicated by this fraction.

2. The *amount* equals the base *plus* the percentage; the *difference* equals the base *minus* the percentage.

What is	*Ans.*		*Ans.*
2. 6% of 950?	57.	**9.** $33\frac{1}{3}$% of 1860 rd.?	620.
3. 8% of 625?	50.	**10.** $66\frac{2}{3}$% of \$3720?	\$2480.
4. 16% of 185?	$29\frac{3}{5}$.	**11.** $42\frac{6}{7}$% of 231 yd.?	99 yd.
5. 25% of 432?	108.	**12.** $45\frac{5}{11}$% of \$165.44?	\$75.20.
6. 35% of 740?	259.	**13.** $\frac{3}{4}$% of $18\frac{3}{4}$ bu.?	$\frac{9}{64}$ bu.
7. $12\frac{1}{2}$% of 512?	64.	**14.** $\frac{4}{5}$% of \$640?	\$5.12.
8. $16\frac{2}{3}$% of 1728?	288.	**15.** $\frac{7}{8}$% of $\frac{27}{28}$ lb. Av.?	.$13\frac{1}{2}$ oz.

16. Find 20% of 54 lb. 11½ oz. Av. *Ans.* 10 lb. 15 1/10 oz.

17. Find 42½% of 13 yd. 1 ft. 8 in. *Ans.* 5 yd. 2 ft. 3⅖ in.

18. In a mixed school of 180 pupils 55 5/9% are girls; how many boys are there? *Ans.* 80.

19. Mr. Smith's salary is $2500 a year, of which he pays 12% for house-rent; what rent does he pay? *Ans.* $300.

20. How much pure silver is there in 412½ oz. of U. S. silver coin if the coin contains 10% alloy? *Ans.* 371.25 oz.

21. If a piece of cloth shrinks 5¾% of its length in sponging, what will be the length after sponging of a piece of 40 yards? *Ans.* 37.7 yards.

22. A barrel of flour weighs 196 lb.; if the bread made from it weighs 33⅓% more than the flour, what is the weight of the bread? *Ans.* 261⅓ lb.

23. A train of cars was running at the rate of 36 miles an hour, when the speed was increased 25%; at what rate did the train then run? *Ans.* 45 miles.

24. If flaxseed contains 11% of oil, and a pint of oil weighs ¾ of a pound, how much linseed oil can be extracted from 2 cwt. 54 lb. of flaxseed? *Ans.* 4 gal. 2 qt. 1.25⅓ pt.

25. A speculator bought 1820 acres of land, of which he sold 25% to Mr. Jones, and 66⅔% of the remainder to Mr. Robinson; how much remained? *Ans.* 455 acres.

26. A lady's income is $3000 a year, of which she spends 15% the first quarter, 12% the second, 12½% the third, and 6¼% the fourth; how much did she spend? *Ans.* $1372.50.

27. A book-keeper has a salary of $3500 a year; he spends 25% for board, 15% for clothes and books, and 12½% for incidentals; what can he save in a year? *Ans.* $1662.50.

28. Mr. Brown owned ⅔ of a factory worth $50,000, and sold 37½% of his share to his partner; what part of the whole factory did he still own, and what was its value?

Ans. 5/12; value, $20,833⅓.

CASE II.

352. Given, the percentage or proceeds and the rate, to find the base.

ORAL EXERCISES.

1. Sixteen is 25 per cent. of what number?

SOLUTION.—If 16 is 25 per cent. of some number, it is $\frac{25}{100}$, or $\frac{1}{4}$, of that number; if 16 is $\frac{1}{4}$ of some number, $\frac{4}{4}$, or the number, equals 4 times 16, or 64.

Find the number of which

2. 14 is 20%.

3. 17 is $16\frac{2}{3}\%$.

4. 28 is $33\frac{1}{3}\%$.

5. 2.7 is 50%.

6. 7 lb. is $6\frac{1}{4}\%$.

7. 3 bu. is $12\frac{1}{2}\%$.

8. 12 A. is $66\frac{2}{3}\%$.

9. 5.5 rd. is 55%.

10. Thirty is 25% more than what number? 50% more than what number? 100% more than what number?

11. Sixty is 25% less than what number? 60 is 50% less than what number? Can 60 be 100% less than any number?

12. A certain village has a population of 5400, which is $12\frac{1}{2}\%$ more than it was three years ago; what is its yearly gain?

13. A gentleman gave his daughter $100 as a Christmas present, which is $62\frac{1}{2}\%$ of what he gave to his wife; what did he give his wife?

14. The average attendance for one quarter of a certain Sunday-school was 240 pupils, which was $11\frac{1}{9}\%$ less than the number registered: how many were registered?

WRITTEN EXERCISES.

1. 40 is 5% of what number? What number, increased by 20% of itself, equals 480?

SOLUTION.—If 40 is 5% of some number, then .05 times *some number* equals 40; if .05 times *some number* equals 40, the *number* equals 40 ÷ .05, which is 800.

OPERATION.

The No. × .05 = 40

The No. = 40 ÷ .05 = 800

SOLUTION.—A number increased by 20%, or .20, of itself, equals 1.20 times the number; and if 1.20 times a *number* equals 480, *the number* equals 480 ÷ 1.20, or 400.

OPERATION.

The No. × 1.20 = 480

The No. = 480 ÷ 1.20 = 400

Or, 100% + 20% = 120%.
120% of the number = 480
Hence 1% of the number = 4
And 100% of the number = 400

Or, 20% = $\frac{1}{5}$; $\frac{5}{5}+\frac{1}{5}=\frac{6}{5}$.
$\frac{6}{5}$ No. = 480
$\frac{1}{5}$ No. = 80
$\frac{5}{5}$ No. = 400

Note.—Teachers can use the solution which they prefer.

Rule.—I. *To find the base, divide the percentage by the rate.*

II. *To find the base, divide the amount by* 1 *plus the rate, or the difference by* 1 *minus the rate.*

Find the number of which

2. 24 is 20%. *Ans.* 120.
3. 32 is 25%. *Ans.* 128.
4. 220 lb. is 40%. *Ans.* 550lb.
5. 723 A. is 30%. *Ans.* 2410 A.
6. $\frac{7}{24}$ is 33⅓%. *Ans.* $\frac{7}{8}$.
7. 18¾ is 75%. *Ans.* 25.
8. $715 is 62½%. *Ans.* $1144.
9. $675¾ is 12½%. *Ans.* $5406.

What number increased by

10. 8% of itself = 54?
11. 12½% of itself = 72?
12. 40% of itself = 1617?
13. 33⅓% of itself = 1728?
14. 16% of itself = $\frac{58}{15}$?
15. 37½% of itself = $\frac{11}{36}$?
16. 60% of itself = $\frac{8}{5}$?
17. 62½% of itself = 9100?

What number diminished by

18. 8⅓% of itself = 55?
19. 18¾% of itself = 91?
20. 25% of itself = 72?
21. 45% of itself = 121?
22. 75% of itself = $\frac{1}{25}$?
23. 87½% of itself = $\frac{8}{81}$?

24. 15 A. 108 P. is 33⅓ per cent. of how many acres of land? *Ans.* 47 A. 4 P.

25. 1 T. 1 cwt. 28 lb. is 16⅔ per cent. more than what number? *Ans.* 18 cwt. 24 lb.

26. A clerk spends 45% of his salary, and can thus save $1100 a year; what is his salary? *Ans.* $2000.

27. A bootblack earns $5, which is 25% of the money he has already put in the savings-bank; how much had he in bank? *Ans.* $20.

28. To pay a note for $6750, I am obliged to draw $42\frac{6}{7}\%$ of my bank deposit; what did I have in bank? *Ans.* $15,750.

29. The population of a city increased 30,000 in 10 years, which was a gain of 20%; what was the population at the beginning of the 10 years? *Ans.* 150,000.

30. A teacher spends 25% of his salary for board, 10% for clothes, 15% for books, and $12\frac{1}{2}\%$ for travelling expenses and incidentals; he saves $375; what is his salary? *Ans.* $1000.

31. A grain-dealer bought a quantity of flour, and sold 20% of it to Mr. Martin and $62\frac{1}{2}\%$ of the remainder to Mr. Mason; if he sold Mr. Mason 850 barrels, how much did he buy? *Ans.* 1700 barrels.

32. A lady drew $33\frac{1}{3}\%$ of her money from bank, and paid $37\frac{1}{2}\%$ of it for a house costing $5700; how much money had she remaining in bank? *Ans.* $30,400.

33. Mr. Green invested 80% of his money in railroad stock, but afterward sold 25% of his stock, and still had $6000 invested; what was the whole amount of his money?

Ans. $10,000.

CASE III.

353. Given the percentage or the proceeds and the base, to find the rate.

ORAL EXERCISES.

1. 15 is what per cent of 60?

SOLUTION.—60 is 100 per cent. of 60, and 15, which is $\frac{1}{4}$ of 60, is $\frac{1}{4}$ of 100 per cent., or 25 per cent. of 60.

What per cent.

2. Of 60 is 12?
3. Of 75 is 25?
4. Of $16 are $4?
5. Of 16 qt. are 3 qt.?
6. Of $\frac{3}{4}$ is $\frac{4}{5}$?
7. Of $\frac{3}{5}$ is $\frac{2}{3}$?
8. Of 75% is 15%?
9. Of 2.5% is 1.5%?

10. From a hogshead of ale, containing 80 gallons, 12 gallons leaked out; what per cent. was lost?

11. A base-ball club won 12 games and lost 8 games; what per cent. of its games did it win?

12. The standard for gold and silver coin in the United States is 9 parts pure to 1 of alloy; what % of pure metal is there?

13. A merchant having invested \$3000 finds, on withdrawing his money, that he has received \$3750; what % did he gain?

WRITTEN EXERCISES.

1. 24 is what per cent. of 96?

Solution.—If 24 is some per cent. of 96, then 96 multiplied by *some rate* equals 24; if 96 multiplied by *some rate* equals 24, the *rate* equals 24 divided by 96, which is .25 or 25%.

OPERATION.

$96 \times \text{rate} = 24$

$\text{Rate} = 24 \div 96 = .25$

2. 280 yd. being increased by a certain per cent. of itself, equals 350 yd.; required the rate.

Solution 1st.

$350 - 280 = 70 =$ the percentage.

Then, $280 \times \text{rate} = 70$

$\text{Rate} = 70 \div 280 = .25 = 25\%$

Solution 2d.

$280 = 100\%$

$1 = \frac{1}{280}$ of 100%

$70 = \frac{70}{280}$ or $\frac{1}{4}$ of $100\% = 25\%$

Rule.—I. *To find the rate, divide the percentage by the base.*

II. *To find the rate, divide the difference between the proceeds and base by the base.*

1. The rate may also be found by dividing the proceeds by the base and taking the difference between 1 and the quotient.
2. Teachers can use the method of analysis which they prefer.

What per cent. of

3. 450 is 90? *Ans.* 20%.

4. 840 is 210? *Ans.* 25%.

5. 900 is 360? *Ans.* 40%.

6. 960 is 512? *Ans.* $53\frac{1}{3}\%$.

7. $\frac{5}{6}$ is $\frac{3}{4}$? *Ans.* 90%.

8. $\frac{8}{9}$ is $\frac{2}{5}$? *Ans.* 45%.

9. 64% is $5\frac{1}{3}\%$? *Ans.* $8\frac{1}{3}\%$.

10. 4.5% is $3\frac{3}{8}\%$? *Ans.* 75%.

11. In a school with 250 pupils registered the average attendance is 175; what is the per cent. of attendance?

Ans. 70%.

12. If a miller takes 8 quarts of every bushel he grinds for toll, what per cent. does he take for toll? *Ans.* 25%.

13. A man walked 25 miles in a week, and rode 75 miles; what per cent. of the entire distance did he walk? *Ans.* 25%.

14. A merchant bought 468 yd. of cloth and sold 312 yd.; what per cent. of his cloth did he sell? *Ans.* $66\frac{2}{3}$%.

15. If the base is given as $28.20, and the percentage $4.70, what is the rate per cent.? *Ans.* $16\frac{2}{3}$%.

16. The gold dollar weighs 25.8 gr., and the alloy in it weighs 2.58 gr.; what % of the coin is alloy? *Ans.* 10%.

17. A cask containing 28 gal. 3 qt. lost 5 gal. 3 qt. by leakage; what per cent. leaked out? *Ans.* 20%.

18. A house worth $5000 rents for $600 a year; what per cent. of the value is the rent? *Ans.* 12%.

19. A regiment lost in a campaign 300 men out of 945; what was the per cent. of loss? *Ans.* $31\frac{47}{63}$%.

20. A merchant's liabilities are $15,760, and his assets $7289; what % of his debts can he pay? *Ans.* $46\frac{1}{4}$%.

21. A farmer sold 24 acres of land, which was 20% of what he had left; what per cent. of his farm did he sell? *Ans.* $16\frac{2}{3}$%.

22. An ice-dealer stored 7500 tons of ice, and after losing 15% of it he sold 60% of the remainder; what per cent. of the whole remained? *Ans.* 34%.

23. A lady put $864 in a savings-bank, which was 25% of all her money, and afterward deposited 45% of the rest of her money; what per cent. of all her money had she then in bank? *Ans.* $58\frac{3}{4}$%.

24. 25 per cent. of a regiment being sick, only 720 men were able to enter battle, of whom $\frac{1}{6}$ were killed; how many did the regiment number, and what per cent. of the whole number were killed? *Ans.* 960 men; $12\frac{1}{2}$%.

GENERAL FORMULAS.

354. These methods and rules may all be represented in general formulas as follows:

CASE I.	CASE II.
1. $\text{Base} \times \text{rate} = \text{Percentage}$.	**1.** $\text{Percentage} \div \text{rate} = \text{base}$.
2. $\text{Base} \times (1 + \text{rate}) = \text{Amount}$.	**2.** $\text{Amount} \div (1 + \text{rate}) = \text{base}$.
3. $\text{Base} \times (1 - \text{rate}) = \text{Difference}$.	**3.** $\text{Difference} \div (1 - \text{rate}) = \text{base}$.

CASE III.

$\text{Percentage} \div \text{base} = \text{rate}$.
$\text{Amount} \div \text{base} = 1 + \text{rate}$.
$\text{Difference} \div \text{base} = 1 - \text{rate}$.

NOTE.—These formulas apply to all the cases in the practical applications, and may be used instead of the rules, or with them, as the teacher prefers.

APPLICATIONS OF PERCENTAGE.

355. The **Applications of Percentage** are extensive, owing to the great convenience of reckoning by the hundred in business transactions.

356. These Applications of Percentage are of two classes; those not involving time and those involving time. The following are the most important of these applications:

1st Class.

1. Profit and Loss.
2. Commission.
3. Stocks, Dividends, etc.
4. Premium and Discount.
5. Brokerage.
6. Stock Investments.
7. Taxes.
8. Duties or Customs.
9. Insurance.

2d Class.

1. Simple Interest.
2. Annual Interest.
3. Promissory Notes.
4. Partial Payments.
5. True Discount.
6. Discounting and Banking.
7. Exchange.
8. Compound Interest.
9. Annuities.

TRADE DISCOUNTS.

357. A **Trade Discount** is a deduction from the price of goods, the amount of a bill, etc.

358. The **List Price** of goods is the fixed or published price. The **Net Price** is the list price minus the discount.

There are often several successive discounts deducted from a bill. Thus, when a bill is marked "20 and 10% off," it means that 20% is to be deducted from the bill, and 10% from the remainder.

Sometimes one or more of the discounts are expressed as fractions; thus, "⅙ and 10% off."

Fixed price lists are printed in certain kinds of business, and the rates of discount are changed as the market prices vary.

WRITTEN EXERCISES.

1. A merchant sold a bill of goods amounting to $600, and marked it "10 and 5% off;" what was the net amount of the bill?

Solution.—10% of $600 is $60, which taken from $600 leaves $540; and 5% of $540 is $27, which taken from $540 leaves $513.

OPERATION.

10% of $600 = $60
$600 − $60 = $540
5% of $540 = $27
$540 − $27 = $513

2. I sold a lot of envelopes marked $7.50 ℔ M., at 10 and 10% off; what was the price received? *Ans.* $6.075.

3. Some valentines marked $15 were sold at 25, 20, and 10% off; what was the selling price? *Ans.* $8.10.

4. Bought 2 doz. cast-steel hammers at $32.06¼ a dozen, at 25 and 5% off; what was the "list" price? *Ans.* $45.

5. Sold a lot of carriage-bolts @ $2.91 ℔ C., which was 10 and 5% on; what was their cost? *Ans.* $2.52.

6. I bought a quantity of Christmas cards, wholesale, at a discount of 50, 50, and 25%; what was the rate of discount? *Ans.* 81¼%.

7. What deduction from price is 10 and 15% off? What deduction is 5, 10, and 15% off? *Ans.* 23½%; 27$\frac{13}{40}$%.

8. What is the difference between 10% on and 5 and 5% on? between 20 and 10% on and 20 and 10% off? *Ans.* ¼%; 60%.

9. Bought a bill of envelopes amounting to $75.20 on 30 days' credit at $\frac{1}{8}$ and 5% off, and 2% off for cash; what did I pay for them? *Ans.* $61.26.

10. Bought a bill of valentines amounting to $175.50 on 3 mo. credit at $\frac{1}{6}$, 10, and 5% off, and 2% for cash; what was the net cash amount of the bill? *Ans.* $122.54.

11. What is the difference in a bill of $750 between a discount of 30 off and a discount of 20 and 10% off? *Ans.* $15.

12. What is the difference between a discount of 40% and 10% taken 4 times? Between 40% and 20% taken twice? *Ans.* 5.61%; 4%.

PROFIT AND LOSS.

359. **Profit and Loss** are terms which denote the gain or loss in business transactions.

360. The **Quantities** considered are as follows:

1. The **Cost**, which is the *base*.
2. The **Rate** of profit or loss.
3. The **Profit** or **Loss**, which is the *percentage*.
4. The **Selling Price**, which is the *amount* or *difference*.

In marking goods it is customary to take one or more words or a phrase or sentence, consisting of ten different letters, and let each letter in succession represent one of the Arabic figures. The prices marked thus can be read only by those who have the key.

CASE I.

361. Given the cost and rate of profit or loss, to find the profit or loss or the selling price.

ORAL EXERCISES.

1. A lady paid $8 for a dress, and sold it at a loss of 25%; required the loss.

SOLUTION.—At a loss of 25 per cent., $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the loss. $\frac{1}{4}$ of $8 is $2. Therefore, etc.

2. A grocer bought coffee at 60 cents a pound, and sold it so as to gain 20%; what was the gain?

3. James paid $75 for a bicycle, but afterward sold it at a loss of 20%; for what did he sell it?

4. A lady bought a velvet coat for $90, and sold it at a gain of 16⅔%; what was her gain?

5. If I buy Henrietta cloth at 40¢ a yard and sell it at an advance of 12½%, what is my selling price?

6. A sofa was sold at auction for $20, and, having been repaired at a cost of 20%, was sold at an advance of 33⅓% on its whole cost; what was the gain?

WRITTEN EXERCISES.

1. A house was bought for $5700, and sold at a gain of 12%; what was the gain?

SOLUTION.—If the house was bought for $5700, and sold at a gain of 12%, the gain was .12 times $5700, which is $684.

OPERATION.

$$\begin{array}{r} \$5700 \\ .12 \\ \hline \$684.00 \end{array}$$

Rule.—*To find the profit or loss, multiply the cost by the rate.*

The selling price equals the cost multiplied by 1 plus the rate of profit, or by 1 minus the rate of loss.

2. A grain-dealer bought wheat at 65¢ a bushel, and sold it at a gain of 7½%; what was the selling price? *Ans.* 69⅞¢.

3. If a coal-dealer buys coal at $4.75 a ton and sells at 18% advance, what is his gain? *Ans.* $.85½.

4. Paid $520 for a lot of dress-goods, and sold them at a gain of 15½%; what did I receive for the goods? *Ans.* $600.60.

5. Mr. Wood weighed 175 lb., but by a spell of sickness he lost $14\frac{2}{7}$% of his weight; what is his weight? *Ans.* 150 lb.

6. A dealer in musical instruments buys pianos at 35% off, and sells at a gain of 30%; what does he get for a piano whose list price is $750? *Ans.* $633.75.

7. A train of cars was running 25 miles an hour, but, wishing to make up lost time, the engineer increased the speed 25%; what was the rate then run? *Ans.* 31¼ miles.

8. A drover bought 80 cows at $20 a head; if 10 were killed by an accident, for how much must he sell the remainder per head to gain 25%? *Ans.* $28⁴⁄₇.

9. A merchant bought 64 yd. of French chally at 45¢ a yard, and sold it at a gain of 33⅓%; what did he gain by the transaction? *Ans.* $9.60.

10. I bought 60 yards of silesia @ 15¢, and marked it at a profit of 20%; what will be my profit if I sell 11⅑% lower than the selling mark? *Ans.* 6⅔%.

11. A gentleman bought a yacht for $4500, sold it at a gain of 25%, and the buyer sold it at a loss of 20%; what did the latter receive? *Ans.* $4500.

12. At the close of the season a merchant marks down a certain class of goods 12½%; what will be the marks of those selling at 16¢, 20¢, 25¢, 37½¢, and 40¢, taking the nearest half-cent? *Ans.* 14¢, 17½¢, 22¢, 33¢, 35¢.

13. The firm of Knox & Irwin use as their key for marking goods, "Samuel Knox;" if they buy cashmere at 50¢ a yard, how must it be marked to sell at a gain of 25%? *Ans.* la $\frac{s}{a}$.

14. Mr. Colton had as his key "don't give up;" if he buys silk at $1.25 a yard, how must he mark it to fall 10% and yet gain 8%? *Ans.* d. gp.

CASE II.

362. Given the rate and the profit or loss, or the selling price, to find the cost.

ORAL EXERCISES.

1. A grocer sold flour for 50¢ a barrel less than cost, and thereby lost 10%; what did it cost him?

SOLUTION.—If he lost 10 per cent., then $\frac{10}{100}$, or $\frac{1}{10}$ of the cost, equals the loss, which equals 50 cents; if $\frac{1}{10}$ of the cost equals 50 cents, $\frac{10}{10}$, or the whole cost, equals 10 times 50 cents, or 5 dollars.

2. A merchant charged 50 cents more for a trunk than it cost him, and thereby gained 10%; what was the cost?

3. Mr. Thompson sold a horse for $91, and thereby lost 35%; what did the horse cost him?

4. A drover bought a cow for 20% less than its value, and sold it for 30% more than its value; what % did he gain?

5. A picture-dealer sold two pictures for $150 each; on one he gained 25%, and on the other he lost 25%; what was his gain or loss?

WRITTEN EXERCISES.

1. A man gained $50 on a watch by selling it at a gain of 20%; what did the watch cost?

SOLUTION.—At a gain of 20%, .20 times the *cost* equals the gain, which is $50; if the *cost* multiplied by .20 equals $50, the *cost* equals $50 divided by .20, or $250.

OPERATION.

Cost × .20 = $50

Cost = $50 ÷ .20 = $250

Or, If 20% of the cost = $50

1% of the cost = $\frac{1}{20}$ of $50 = $2½

100% of the cost = $2½ × 100 = $250

Or, 20% = $\frac{1}{5}$

$\frac{1}{5}$ of cost = $50

$\frac{5}{5}$ of cost = $250

Rule.—*To find the cost, divide the profit or loss by the rate.*

The cost equals the selling price divided by 1 plus the rate of profit, or by 1 minus the rate of loss.

2. I sold velvet at $2.75 a yard, and gained 10%; what did the velvet cost me? *Ans.* $2.50.

3. A furrier sold a damaged sealskin coat for $175, and lost 12½%; what was the cost? *Ans.* $200.

4. A drover lost 2½% of his cattle by disease and 3½% by accident; his whole loss was 42; how many had he at first? *Ans.* 700.

5. I sell ginghams @ 15¢ and make 20%, and another lot @ 12¢ and lose 20%; what was their cost? *Ans.* 12½¢; 15¢.

6. On silks marked $2.50 I can fall 10% on my price and make 12½% on the cost; what was the cost? *Ans.* $2.00.

7. A manufacturer's capital produces an income of \$5120 in a year at a gain of 16%; what would be his income at a gain of 24%? *Ans.* \$7680.

8. Prof. Allen sells his library for \$1000 less than it cost him, and thereby loses 20%; what would he have received if he had sold it at an advance of 20%? *Ans.* \$6000.

9. A case of dry goods was somewhat damaged when opened; I marked them 76¢ a yard, so that I can abate 25%, and yet lose only 5%; what did they cost me? *Ans.* 60¢.

10. A merchant, having some goods marked 22¢, marked them down $9\frac{1}{11}$%, and still had a profit of 25%; what did they cost him? *Ans.* 16¢.

11. Mr. Walcott's key for marking goods is "Charleston;" if he has satin marked *a.ln*, and gains at that $16\frac{2}{3}$%, what was the cost? *Ans.* \$3.00.

12. Mr. Winslow offered his house for sale at an advance of 20%, but afterward sold it for \$6300, which was $12\frac{1}{2}$% less than his original offer; what was the first cost of the house? *Ans.* \$6000.

CASE III.

363. Given the cost and the profit or loss or the selling price, to find the rate.

ORAL EXERCISES.

1. A lady bought a piece of land for \$120, and sold it at a gain of \$15; what was her gain per cent.?

SOLUTION.—If on \$120 she gained \$15, on \$1 she gained $\frac{1}{120}$ of \$15, which is \$$\frac{15}{120}$, or \$$\frac{1}{8}$, and on \$100 she would gain 100 times \$$\frac{1}{8}$, which are \$$\frac{100}{8}$, or \$$12\frac{1}{2}$; hence the gain is $12\frac{1}{2}$ per cent.

2. If a grocer buys flour @ \$4, and sells it at a gain of \$1.60, what is his gain per cent.?

3. Sold a lot which cost \$700 for \$525; what was the loss per cent.?

4. What % is gained by selling goods at *double* their cost? What % is lost by selling them at *one-half* their cost?

5. If a miller takes 3⅕ quarts out of every bushel he grinds for toll, what % does he take for toll?

6. If goods are bought at 20% below market price, and sold at 16% above that price, what is the gain %?

7. I bought a lot of goods at 5% below current prices, and sold them at 5% above those prices; what was my gain %?

8. A coal-dealer sold some coal for $18, and thereby cleared ⅙ of this money; what would he have lost % if he had sold it for $12?

WRITTEN EXERCISES.

1. A man bought a watch for $80, and sold it at a loss of $8; what was the loss per cent.?

Solution.—Since $80, the *base*, multiplied by the *rate* of loss, equals $8, the rate must equal $8 divided by $80, which is .10, or 10%.

OPERATION.

$80 × rate = $8

Rate = $8 ÷ $80 = .10

Rule.—*To find the rate, divide the profit or loss by the cost.*

The rate equals the difference between the cost and the selling price, divided by the cost.

2. If milk is bought at 3¢ a quart and sold at 5¢, what is the gain %? *Ans.* 66⅔%.

3. If a grocer buys coffee for 50¢ a lb. and sells @ 65¢, what does he gain %? *Ans.* 30%.

4. A merchant sold damaged goods costing 50¢ a yard for 35¢; what was the loss per cent.? *Ans.* 30%.

5. Bought a lot of goods for 20% below the market price, and sold them for 20% above market price; what % did I gain? *Ans.* 50%.

6. A stationer bought paper at $2.50 a ream, and sold it at 16¢ a quire; what % did he gain? *Ans.* 28%.

7. If I buy at 20 and 10% off, and sell for 20 and 10% on, what % do I gain? *Ans.* 83⅓%.

8. Bought Easter cards at 25, 20, and 10% off, and sold for *10 and 5% on*; what per cent. did I gain? *Ans.* 113⅞%.

9. A real-estate dealer, having a tract of land, sold 20% to A, and 75% of the remainder to B; what % of the whole remained? *Ans.* 20%.

10. Mr. Sansom bought 60 shares of bank-stock for $6600, and sold 40 shares for what they all cost; what was the gain %? *Ans.* 50%.

11. Miss Hartman sold her piano for $450, clearing 20% of this amount; what would have been her gain % if she had received $30 more? *Ans.* 33⅓%.

12. A merchant marked a lot of goods *u. bn*, his key being "Republican;" if he bought them @ $3.60, what was his gain % at the marked price? *Ans.* 25%.

13. I paid $150 for a watch, and asked such a price that after falling $20 I still made 20% on the cost; what % did I abate from the asking price? *Ans.* 10%.

COMMISSION.

364. **Commission** is a percentage paid to a person who transacts business for another.

365. The person who transacts business for another is called a *Commission Merchant, Agent,* or *Broker.*

366. The merchandise sent to an agent to be sold is called a *Consignment.*

367. The person who sends the merchandise is called the *Consignor;* the person to whom the merchandise is sent is called the *Consignee.*

368. The **Net Proceeds** is the sum left after the commission and charges have been deducted from the amount of a sale or collection.

369. The **Entire Cost** is the sum obtained by adding the commission and charges to the amount of a purchase.

370. The **Quantities** considered are: 1. The *amount sold, bought,* etc.; 2. The *Rate of Commission;* 3. The *Commission;* 4. The *Entire Cost* or *Net Proceeds.*

371. The **Base** in commission is the actual *amount of the sale, purchase, collection,* or *exchange.*

An agent residing at a great distance from his employer is often called a *correspondent;* the person for whom an agent does business is called the *principal.*

CASE I.

372. Given the base and rate, to find the commission, or the net proceeds, or the entire cost.

1. An agent sold goods to the amount of $7650, his rate of commission being 3½%; what was his commission?

Solution.—At the rate of 3½% the commission was .03½ times $7650, which equals $267.75.

OPERATION.

$$\begin{array}{r} \$7650 \\ \underline{.03\tfrac{1}{2}} \\ \$267.75 \end{array}$$

WRITTEN EXERCISES.

2. An agent sold goods to the amount of $8725; what was his commission at 2½%? *Ans.* $218.12½.

3. An agent collects $3840 on a commission of 3½%; how much should he remit to his employer? *Ans.* $3705.60.

4. An auctioneer sold a house at auction for $5784, and the furniture for $3250.85; what was his commission at 2¼ per cent.? *Ans.* $203.28.

5. An agent sold 350 acres of land at $25 an acre on a commission of 5%; required the commission and the amount paid over. *Ans.* $437.50; $8312.50.

6. What would be the net proceeds of a consignment of 500 bbl. of flour, @ $4.25, allowing 2¼% commission, and paying storage for 15 days at 5¢ a barrel? *Ans.* $2052.18¾.

7. A debt of $1728 is compromised for 85%; what is the collector's commission at 4½%, and what are the proceeds remitted? *Ans.* $1402.70.

8. A factor sells $6742 worth of woolen goods, charging 2½% commission and 1¼% for insuring payment; what amount will he remit to his principal? *Ans.* $6489.17½.

9. A tax-collector had a warrant for $35,870, $15,000 of which he collected at 1½%, and the remainder at 2%; what was the amount of his fees? *Ans.* $642.40.

10. A college made a contract with an architect to erect a library-building to cost $50,000, and allowed him ¾% for his plans and specifications and 2¼% for superintendence; when the accounts were settled he claimed $1550; how much did he overcharge? *Ans.* $50.

11. A factor charges 5¾% when he guarantees payment, and 4% when he does not. He receives a consignment of cotton which he sells for $9725, but $325 of the amount turns out a bad debt; which rate of commission on this transaction would be the most profitable for the agent?

Ans. The second, by $154.81¼.

CASE II.

373. Given the rate and the commission, or the net proceeds, or the entire cost, to find the base.

1. An agent receives $302.75 commission for buying a house, at the rate of 3½%; what was the cost of the house?

SOLUTION.—At a commission of 3½%, .03½ times the cost of the house equals the commission, which is $302.75; hence, the *cost* equals $302.75 divided by .03½, which we find is $8650.

OPERATION.

$$\frac{\$302.75}{.03\frac{1}{2}}=\$8650$$

2. An agent receives $5535 to be invested in goods after retaining his commission, 2½%; required the amount invested.

SOLUTION.—The sum to be invested, increased by 2½% of itself, equals 1.02½ times the sum, which equals $5535. If 1.02½ times the *sum* equals $5535, the sum equals $5535 divided by 1.02½, which we find is $5400.

OPERATION.

$$\frac{\$5535}{1.02\frac{1}{2}}=\$5400$$

WRITTEN EXERCISES.

3. An agent received $275 for collecting rents; if his commission was 5%, what was the amount collected? *Ans.* $5500.

4. A cotton-factor received $7210 to be invested in cotton after deducting his commission of 3%; how much was his commission? *Ans.* $210.

5. An agent buys a quantity of leather, paying $20 for cartage, and charging 1¼% commission; the entire cost was $7613.75; what was the commission? *Ans.* $93.75.

6. A consignment of cotton was sold on commission, the agent charging 1¾% for the sale and 2½% for guaranteeing payment; the commission was $286.87½; what was the amount received for the goods? *Ans.* $6750.

CASE III.

374. Given the base and the commission, or the net proceeds, or the entire cost, to find the rate.

1. An agent collects $6300, and his commission was $84; required the rate of commission.

SOLUTION.—The commission, $84, equals the sum collected, $6300, multiplied by the *rate;* hence, the *rate* equals $84 divided by $6300, which we find is .01⅓ or 1⅓%.

OPERATION.

$$\frac{\$84}{\$6300}=.01\tfrac{1}{3}$$

WRITTEN EXERCISES.

2. An agent in Kansas sold some land for me, and, retaining his commission, $180, remitted me $3820; what was the rate of commission? *Ans.* 4½%.

3. My agent bought 50 horses, @ $150, paid $25 for keeping and $80 for transportation; he drew on me for $7867.50; what was the rate of commission? *Ans.* 3½%.

4. I sold a consignment of flour through an agent for $19,400; my commission was $970, and I paid the agent $582; what was the rate of commission of each? *Ans.* My com. 5%; agent's, 3%.

5. A commission-merchant sold a consignment of wheat for $15,000; he charged $225 for commission and $375 for guaranteeing payment; what was the rate of commission and of *guaranty?* *Ans.* Com., 1½%; guar., 2½%.

STOCKS AND DIVIDENDS.

375. A **Company** is an association of individuals for the transaction of business.

376. A **Corporation** is a company regulated in its operations by a general law or a special charter.

377. The capital invested in the business of a company is called the **Stock.** The owners of stock are called *Stockholders.*

378. The stock is divided into equal parts called **Shares.** A share is usually $50 or $100.

379. An **Installment** is a sum required of stockholders as a payment on their subscription.

380. A **Dividend** is a sum paid to stockholders out of the gains of the company.

381. An **Assessment** is a sum required of stockholders to meet the expenditures or losses of the company.

1. *Bonds* are written or printed obligations to pay certain sums of money at or before a specified time.

2. *State* or *United States Bonds* are bonds of a State or of the United States, payable at some future time, with interest at a fixed rate.

3. *Stocks* is a general name applied to the scrip or bonds of a corporation and to government bonds and public securities.

4. *Certificates of Stock* are the papers issued by a corporation to its stockholders as evidence of the number of shares belonging to each respectively.

382. The **Base** upon which dividends and assessments are estimated is the original or par value of the stock.

383. The **Quantities** considered are as follows: 1. The *Stock;* 2. The *Rate;* 3. The *Dividend* or *Assessment.*

NOTE.—In order to make these business subjects intelligible, teachers should show pupils the business papers, illustrate how a company is formed, its stock issued, its dividends declared and paid, etc. The subject is simple when pupils have a clear idea of the real business transactions referred to in the examples.

CASE I.

384. Given the stock and rate of dividend or assessment, to find the dividend or assessment.

1. A owns 300 shares, at $50 each, of the stock of a bank which declares a dividend of 6%; what is his dividend?

SOLUTION.	OPERATION.
SOLUTION.—A's capital is $50 × 300 = $15,000, and his dividend equals $15,000 × .06 = $900.	$50 × 300 = $15,000 $15,000 × .06 = $900

Principle.—*The dividend or assessment equals the par value of the stock multiplied by the rate.*

NOTE.—It is often convenient to find the result by multiplying the dividend or assessment on one share by the number of shares.

WRITTEN EXERCISES.

2. What is the amount of a 5% dividend on 65 shares of stock, par $50? *Ans.* $162.50.

3. A lady owns 75 shares of Penna. R. R. stock, par $50; if the company declares a semi-annual dividend of 3%, what does she receive? *Ans.* $112.50.

4. Mr. Garrett has 85 shares, at $50, in the Reading R. R. Co., which, on account of reorganization, calls for an assessment of 4½%; what does he pay? *Ans.* $191.25.

5. A telephone company whose stock is $750,000 declares a semi-annual dividend of 5%; what is the amount of the dividend? *Ans.* $37,500.

6. I have 80 shares ($50) of stock in a bank which declares a dividend of 4%; what is my dividend, and how many shares of stock would it buy at par? *Ans.* 3 shares and $10 surplus.

7. Mr. Stanton owns 120 shares in an Electric Traction Co., par $50, which declares an extra dividend of 10%, payable in stock; how many shares will he then own? *Ans.* 132 shares.

8. An electric lighting company, having $500,000 capital, pays a dividend of $33.75 on 15 shares ($50), reserving as a *surplus* $7500; what were the net earnings? *Ans.* $30,000.

CASE II.

385. Given the rate and the dividend or assessment, to find the stock.

1. A bank divides $7500 among the stockholders, being the amount of a 4% dividend; required the whole amount of stock.

SOLUTION.—If $7500 is 4% of the stock, then .04 times the *stock* equals $7500; hence, the *stock* equals $7500 divided by .04, which is $187,500.

OPERATION.

$$\frac{\$7500}{.04} = \$187,500$$

Principle.—*The par value equals the dividend or assessment divided by the rate.*

WRITTEN EXERCISES.

2. I received $875 from a 3½% dividend; what is the face value of my stock? *Ans.* $25,000.

3. I receive $742 as my share of a 7% dividend; how many shares at $50 each do I own? *Ans.* 212 shares.

4. I paid an assessment of $350, at 3½%, on insurance stock; how many shares, $50 each, had I? *Ans.* 200 shares.

5. A trolley company divides $15,000 among its stockholders, being a 5% dividend; what is C's stock if he owns $\frac{1}{10}$ of the whole? *Ans.* $30,000.

6. An electric lighting company whose annual expenses are $5780, and gross earnings $8280, pays a dividend of 5%; required the face value of the stock. *Ans.* $50,000.

7. A lady received $500, payable in stock, as a 10% dividend; how many shares had she at first, and how many has she now, shares $100 each? *Ans.* 50; 55.

8. Mr. Allen received 4 shares and $40 in money as a 6% dividend; how many shares, valued at $50 each, did he then own? *Ans.* 84 shares.

9. I received a stock dividend of 20% in the Pennsylvania R. R., and I then had 108 shares; how many shares, @ $50 each, had I at first? *Ans.* 90 share

10. I received two stock dividends in a mining company, the first of 8% and the second of 10%; I then owned 297 shares, at $25 a share; how many had I at first? *Ans.* 250 shares.

CASE III.

386. Given the stock and dividend or assessment, to find the rate.

1. A mining company, whose stock is $500,000, clears $37,500 in a year; what rate of dividend can it declare?

SOLUTION.—Since the dividend is some per cent. of the stock, the base, $500,000, multiplied by the *rate*, equals $37,500; hence, the *rate* equals $37,500 divided by $500,000, which equals .07½.

OPERATION.

$$\frac{\$37,500}{\$500,000} = .07\tfrac{1}{2}$$

WRITTEN EXERCISES.

2. A company whose stock is $150,000 requires an assessment of $3750; what was the rate? *Ans.* 2½%.

3. Mr. A owns 85 shares of railroad stock, @ $50, and receives a dividend of $233.75; what was the rate? *Ans.* 5½%.

4. Mr. Thomson owns 80 shares ($100) in a bank whose capital is $250,000; he receives a dividend of $480; what is the rate of dividend and the whole dividend? *Ans.* 6%; $15,000.

5. A life and trust company, having a capital of $1,500,000, clears $50,000; what is the largest integral rate of dividend they can declare, and what is the surplus? *Ans.* 3%; $5000 surplus.

6. After receiving a stock dividend I had 80 shares ($100), and $64 toward another share; if I had 72 shares at first, what was the rate of dividend? *Ans.* 12%.

7. A lady had 200 shares in a Colorado mining company ($25), and received a stock dividend of 10 shares one year, and the next year of 16 shares and $20; what were the rates of dividend? *Ans.* 5%; 8%.

PREMIUM AND DISCOUNT.

387. The **Par Value** of stock is the value named in the certificate, called the *nominal value* or *face.*

388. The **Real Value,** or *Market Value,* of stock is the amount for which it will sell.

389. Stock is **Above Par,** or at a *premium* or *advance,* when it sells for more than its nominal value.

390. Stock is **Below Par,** or at a *discount,* when it sells for less than its nominal value.

The stock of a company will generally be above par when the company is doing a lucrative business, and below par when it is doing a poor business. The bonds of a town, city, etc. vary according to the confidence in their security, the fluctuations of the money market, etc.

391. A **Draft,** *Check,* or *Bill of Exchange* is a written order for the payment of money at some definite place.

392. The **Base** upon which premium and discount are estimated is the *par value.*

393. The **Quantities** considered are four: 1. The *Par Value;* 2. The *Rate;* 3. The *Premium* or *Discount;* 4. The *Real Value.*

The problems under this subject are solved without brokerage—the sales and exchanges being regarded as direct without the aid of a broker.

CASE I.

394. Given the par value and the rate of premium or discount, to find the premium or discount or real value.

1. A broker bought 25 shares of stock ($100) at 6% premium; required the premium and cost or real value.

Solution.—The par value of 25 shares at $100 each is $100 × 25 = $2500, and the premium at 6% is .06 times $2500, which is $150, and this, added to the par value, equals $2650, the real value.

OPERATION.

$100 × 25 = $2500 par value.
.06
$150.00 premium.
2500.00
$2650.00 real value.

Principle.—*The premium or discount equals the par value multiplied by the rate.*

WRITTEN EXERCISES.

2. I sold 54 shares of gas-stock ($100) at 3% discount; required the discount and the real value. *Ans.* $162; $5238.

3. When gold was at a premium of 16%, how much in greenbacks was required to buy $640 in gold? *Ans.* $742.40.

4. What is the market value of 65 shares of N. J. Central ($100) at 8% below par? *Ans.* $5980.

5. A broker bought 45 shares ($100) N. Y. Central at $\frac{1}{2}$% discount, and sold them at $2\frac{1}{4}$% premium; what did he gain in the transaction? *Ans.* $123.75.

6. Mr. Reed bought $2000 Phila. & Erie 5's at $17\frac{1}{2}$% premium, and paid $2000 in drafts, at $1\frac{1}{4}$% discount, and the balance in cash; how much cash did he pay? *Ans.* $375.

7. A banker exchanged 80 shares ($50) Lehigh Valley R. R., at a discount of $3\frac{1}{2}$%, for 30 shares ($100) N. Y. Car Trust, at $2\frac{1}{4}$% premium, receiving the difference in cash; how much cash did he receive? *Ans.* $792.50.

CASE II.

395. Given the rate and the premium or discount, or the real value, to find the par value.

1. A broker sold some drafts at a discount of $3\frac{1}{4}$%, receiving $130 less than their face; what was their face value?

SOLUTION.—If the discount at $3\frac{1}{4}$% is $130, then $.03\frac{1}{4}$ times the *par value* equals $130; hence, the *par value* equals $130 divided by $.03\frac{1}{4}$, which we find is $4000.

OPERATION.

$\$130 \div .03\frac{1}{4} = \$4000$

Principle.—*The par value equals the premium or discount divided by the rate.*

WRITTEN EXERCISES.

2. I sold 50 shares of stock at a premium of $5\frac{3}{4}$%, and received $287.50 advance; what was the par value of a share? *Ans.* $100.

3. A broker sold some stocks at 6¾% premium, receiving $337.50 above par; what was their par value? *Ans.* $5000.

4. The premium on a village bond at 1¼% was $3.75; required the face of the bond. *Ans.* $300.

5. If Pennsylvania State 4's are at 15% premium, what is the par value of the amount that can be bought for $4600? *Ans.* $4000.

6. A lady paid $3067.50 for a New York State bond, at 2¼% premium; required its face value. *Ans.* $3000.

7. My broker sold stock bought at par at a premium of 2⅛%, and received $102 advance; how many shares ($50) did he sell? *Ans.* 96 shares.

8. Required the face of a draft, at ½% discount, which will buy 50 shares West Jersey R. R. ($50), selling at 5% premium. *Ans.* $2638.19.

CASE III.

396. Given the par value and the real value, or the premium or discount, to find the rate.

1. A lady sold a draft for $750, at a premium of $26.25; what was the rate of premium?

SOLUTION.—Since the premium equals the par value multiplied by the rate, $750 multiplied by the *rate*, equals $26.25; hence, the *rate* equals $26.25 divided by $750, which we find is .03½.

OPERATION.

$$\frac{\$26.25}{\$750} = .03\tfrac{1}{2}$$

Principle.—*The rate equals the premium or discount divided by the par value.*

WRITTEN EXERCISES.

2. A man bought a draft, face value $2500, at a discount of $12.50; required the rate of discount. *Ans.* ½%.

3. A broker bought 150 shares of railroad stock ($50) for $7050; what was the rate of discount? *Ans.* 6%.

4. If the 150 shares mentioned in the previous example are sold for $7312.50, what is the rate of discount and also the rate of gain? *Ans.* $2\frac{1}{2}\%$; $3\frac{34}{47}\%$.

5. During the Civil War a broker gave a draft for $6250, at 4% discount, for $5000 in gold; what was the premium on gold? *Ans.* 20%.

6. A banker purchased a $5000 bond, premium $12\frac{1}{2}\%$, and sold it for $6000; for what premium did he sell the bond, and what was the gain %? *Ans.* 20% premium; $6\frac{2}{3}\%$ gain.

BROKERAGE.

397. **Brokerage** is a percentage charged by brokers for the transaction of business.

398. A **Broker** is a person who buys or sells money, stocks, bills of exchange, real estate, etc. for others.

A *stock broker* is one who deals in stocks; he is generally called simply a *broker*.

399. The **Rate** of brokerage is usually $\frac{1}{4}\%$, and will be so understood when no other rate is mentioned. In New York the rate is $\frac{1}{8}\%$ on both bonds and stocks.

400. The **Base** upon which the commission for the purchase and sale of bonds and stocks is estimated is their *par value.*

401. The **Quantities** considered are: 1. The *Par value of the amount bought or sold;* 2. The *Rate of Brokerage;* 3. The *Brokerage;* 4. The *Market value of the stock;* 5. The *Entire Cost* or *Net Proceeds.*

1. Stocks are quoted either at the price of one share or at the price of $100 of par value of the stock, whatever be the par value of a share. The former method is used in Philadelphia; the latter in New York.

2. Bonds are often named from the rate of interest they draw; thus, we have 4's, $4\frac{1}{2}$'s, 5's, etc. The time to run or date when due sometimes gives the name; as, 4's of '97.

CASE I.

402. Given the par value, the rate, and the market value, to find the brokerage, net proceeds, or entire cost.

1. A broker bought for me 24 shares Un. Cos. N. J. ($100), rate of brokerage $\frac{1}{4}$% ; required the brokerage.

SOLUTION.—The par value was 24×$100, or $2400; the brokerage was .00$\frac{1}{4}$ times $2400, which equals $6.

OPERATION.
$100 × 24 = $2400
$2400 × .00$\frac{1}{4}$ = $6

Principle.—*The brokerage equals the par value multiplied by the rate.*

NOTE.—It is often shorter to multiply the brokerage on one share by the number of shares. When the par is $50, one-half the rate should be used in this method.

WRITTEN EXERCISES.

2. A broker bought for me 75 shares of bank stock ($100) at par, brokerage $\frac{1}{4}$% ; required the brokerage and the cost of the stock. *Ans.* $18.75.

3. My broker bought for me 150 shares of canal stock ($50), at 44$\frac{1}{4}$; what did the stock cost me, brokerage $\frac{1}{4}$ per cent.? *Ans.* $6656.25.

4. A speculator sold, through his broker, 72 shares Minehill R. R. ($50), at 66$\frac{3}{8}$; required the brokerage and net proceeds. *Ans.* Proceeds, $4770.

5. My broker bought, on my account, 45 shares Lake Shore R. R. ($100), at 112, and sold them at 120; what was his commission at $\frac{1}{8}$% and my profit? *Ans.* Profit, $348.75.

6. Sold 45 shares Norristown R. R. stock ($50) at 105, and bought $4000 Lehigh Valley 6's at 111$\frac{1}{2}$; what balance shall I have in hand from the transaction, including brokerage in both cases? *Ans.* 249.37\frac{1}{2}$.

7. What are the proceeds of the following stocks sold through a broker: 250 shares Texas & Pacific, at 50$\frac{5}{8}$ ($100); 300 shares Morris & Essex ($50), at 60$\frac{3}{4}$; and 200 shares Union Pacific ($100), at 117$\frac{3}{8}$, brokerage $\frac{1}{8}$% ? *Ans.* $54281.25.

CASE II.

403. Given the rate, the brokerage, or the net proceeds, or entire cost, and the market value, to find the par value.

1. My broker charged $75 for selling some drafts for me, at the rate of 1½%; what amount of drafts did he sell?

Solution.—At a rate of 1½%, .015 times the par value of the drafts equals the brokerage, which is $75; hence, the *par value* equals $75 divided by .015, which we find is $5000.

OPERATION.

$$\frac{\$75}{.015}=\$5000$$

Principle.—*The par value equals the brokerage divided by the rate.*

WRITTEN EXERCISES.

2. A broker charged $25 at ¼% for buying Pennsylvania R. R. ($50); how many shares did he buy? *Ans.* 200 shares.

3. I paid my broker $2347.50 for an investment in Reading 4's, at 78, including his commission; what was the par value of the bonds? *Ans.* $3000.

4. I sent a New York broker a draft on Brown Brothers for $4631.25, to pay for Illinois Central ($100) at 92½, and his commission at ⅛%; how many shares did he buy? *Ans.* 50 shares.

5. Through my broker I sold $7000 Penna. State 4's at 115, and invested the proceeds in Northern Central R. R. ($50), at 69½; how many shares did he buy, brokerage on each transaction ¼%? *Ans.* 115 shares; $25.62½ surplus.

CASE III.

404. Given the par value, and the brokerage, or the net proceeds, or entire cost, and the market value, to find the rate.

1. A broker bought City of Pittsburg 7's, par value $8500; his charge was $21.25; what was the rate of brokerage?

Solution.—The brokerage, $21.25, equals the par value, $8500, multiplied by the *rate*; hence, the *rate* equals $21.25 divided by $8500, which we find is .00¼, or ¼%.

OPERATION.

$$\frac{\$21.25}{\$8500}=.00\tfrac{1}{4}$$

Principle.—*The rate equals the brokerage divided by the par value.*

WRITTEN EXERCISES.

2. A broker buys for me 75 shares Philadelphia Traction stock ($50); the brokerage was $9.37½; what was the rate of brokerage? *Ans.* ¼%.

3. My broker having purchased, according to order, 20 shares of United Cos. of New Jersey ($100), at 236½, notifies me that the entire cost is $4735; what is the rate of brokerage? *Ans.* ¼%.

4. I send a broker in New York $10,815, for which he buys 60 shares in the Equitable Gaslight Co., at 180 ($100), retaining the brokerage, and remitting a balance of $7.50; what is the rate? *Ans.* ⅛%.

INCOME FROM INVESTMENTS.

405. **Investments** in stocks may be made either for interest on the money or for gain in selling.

406. The **Several Classes** of stocks, bonds, etc. are those of *Corporations, States,* and the *General Government.*

407. **Bonds** are distinguished as *Registered* and *Coupon Bonds.* The *Registered* bonds are payable to order, and cannot be transferred without being indorsed.

408. The **Coupon** bonds have certificates of interest called coupons attached to them. These coupons may be cut off and presented for payment when the interest is due.

409. The principal bonds of the United States, called Government Bonds, are the following:

The 4's of 1907, which are 4% bonds due in 1907. These bonds are both coupon and registered, and the interest is payable quarterly.

The currency 6's, issued to aid in constructing several railroads to the Pacific. There are several series of these bonds, maturing respectively in 1895, 1896, 1897, 1898, and 1899.

Government Bonds are so secure that they are much sought after for investment, and thus command a premium. This premium usually becomes smaller each year as the bonds approach maturity.

410. A **Mortgage** is a conditional conveyance of property as security for the payment of a debt.

1. Mortgages are usually given to secure the payment of an interest-bearing note. Should the interest not be promptly paid, the mortgage may be *foreclosed*, and the property is then sold by the sheriff to the highest bidder, and the mortgage paid off from the proceeds.

2. Property is usually not mortgaged beyond a certain part of its value, in order that the mortgagee may be secure from loss. A second mortgage is sometimes given on a property, but second mortgages are not always a good security, since in case of foreclosure the first mortgage has a prior claim.

411. A **Ground-Rent** is a fixed rent paid for ground, generally used for building purposes.

It is a common practice in some cities, when a person wishes to build one or more houses, instead of *buying* the ground required, to agree to pay the interest of its value as rent, the contract to continue in force as long as the rent is regularly paid.

NOTE.—In changing from one investment to another there is often a little more realized from the sale of the first than will procure an exact number of shares of the second. In such cases the income will be calculated on the number of shares, without noticing the surplus.

412. The **Quantities** considered are: 1. The *Amount Invested;* 2. The *Rate of Dividend or Interest;* 3. The *Income;* 4. The *Market Value of* $100, *or of one share;* 5. The *Rate of Income.*

CASE I.

413. Given the amount of an investment, the market value, and the rate of dividend or interest, to find the income.

1. A lady invests $5895 in 6% bonds, at 98, brokerage ¼%; what will be her annual income from them?

SOLUTION.—Since for $1 of bonds you pay 98¼ cents, for $5895 you can buy as many dollars' worth of bonds as $.98¼ is contained times in $5895, or $6000. The annual income on the bonds = $6000 × .06, which equals $360.

OPERATION.

$5895 ÷ .98¼ = $6000

$6000 × .06 = $360

WRITTEN EXERCISES.

2. I sent my broker $5662.50 to invest in City of Pittsburg 7's, at 113, brokerage ¼%; what annual income shall I receive? *Ans.* $350.

3. I invested $2556.25 in Pennsylvania R. R. stock ($50), at 51, dividend 5½%, brokerage ¼%; what was my annual income from the investment? *Ans.* $137.50.

4. Mr. Wilson invested $5237.50 in Reading 6's at 104½, interest payable semi-annually, brokerage ¼%; what will be his semi-annual income from the investment? *Ans.* $150.

5. Mr. Jenkins bought on ground-rent a lot 150 ft. front by 210 ft. deep, valued at $56.25 per foot front; what would be the ground-rent per foot front at 6%? *Ans.* $3.37½.

6. A conveyancer sold a lot 50 ft. front and 150 ft. deep on ground-rent, redeemable on payment of $3000; what is the ground-rent at 6% per annum? *Ans.* $180.

7. I have $8500 6% bonds selling at 111; will my income be greater or less if I sell them and buy 7's at 116, brokerage on both transactions ¼%? *Ans.* Greater by $50.

CASE II.

414. Given the income, the rate of dividend, and the market value, to find the amount invested.

1. When 6% railroad bonds are selling at 116, how much must be invested to produce an income of $750?

SOLUTION.—Since $1 of stock gives an income of $.06, to give an income of $750 it will require $750÷.06, or $12500; $12500 of stock at 116% will cost $12500×1.16, or $14500.

OPERATION.

$750 ÷ .06 = $12,500

$12500 × 1.16 = $14,500

WRITTEN EXERCISES.

2. What sum must I invest in 5% bonds, at 89½, to secure an annual income of $800, brokerage ¼%? *Ans.* $14,360.

3. How much must a lady invest in railroad 7's, at 129, to yield her \$350 a year, brokerage $\frac{1}{4}$%? *Ans.* \$6462.50.

4. A conveyancer buys a 6% ground-rent of \$450 per annum at par; what does it cost him? *Ans.* \$7500.

5. A dwelling-house subject to a ground-rent of \$225 at 6% was sold for \$6250; what was its value? *Ans.* \$10000.

6. What sum must be invested in Reading 5's, selling at 119$\frac{1}{2}$, to secure an income of \$800, brokerage $\frac{1}{4}$%? *Ans.* \$19160.

7. What must be the market value of 4% bonds to realize 5% on the investment? Of 7's to realize 5%? *Ans.* 80%; 140%.

8. A man sold \$3500 Government 6's of '98 at 108, and bought sufficient Am. Steamship Co. 6's at 102$\frac{1}{2}$ to yield \$210 income; how much had he left, brokerage $\frac{1}{4}$% on sale and purchase? *Ans.* \$175.

CASE III.

415. Given the market value and the income or rate of dividend, to find the rate of interest on the investment.

1. If I buy 5% bonds at 98, what per cent. of income shall I realize?

SOLUTION.—\$1 of stock will cost \$.98, and pays \$.05; if on \$.98 the gain is \$.05, on \$1 it is as many per cent. as .05 ÷ .98, or 5$\frac{5}{49}$%.

OPERATION.
$.05 \div .98 = .05\frac{5}{49}$

WRITTEN EXERCISES.

2. If a broker buys Lehigh Valley 6's at 124, what rate of income do they pay? *Ans.* 4$\frac{26}{31}$%.

3. When 7.3 bonds are selling at 123, what per cent. will these bonds yield? *Ans.* 5.93+ %.

4. A lady bought a ground-rent of \$78 per annum for \$1400; what % does she realize on her money? *Ans.* 5$\frac{4}{7}$%.

5. Which is the better investment, 5% bonds at 99$\frac{1}{2}$, or 4% bonds at 75$\frac{3}{8}$? 6's at 130, or 4's at 110? *Ans.* 4's; 6's.

6. Mr. Johnson bought Reading R. R. 7's at 115⅜, and afterward exchanged them for Lehigh Navigation 6's at 109½; which investment gives the better income? *Ans.* The 7's.

7. Miss Atherton, desiring a permanent investment, was recommended to take 5's at 108, 6's at 124, or 7's at 129; which gives the better income? *Ans.* The 7's.

GENERAL TAXES.

416. A **Tax** is a sum of money assessed on persons or property for public purposes.

417. A **Property Tax** is a tax upon property. Property is of two kinds—*Real Estate* and *Personal Property.*

418. **Real Estate** is immovable property; as, land, buildings, etc. *Personal Property* is movable property; as, money, stock, furniture, etc.

419. A **Poll Tax** is a fixed sum assessed on each person or citizen of a community.

420. A list or schedule containing the names of persons taxed, the valuation of their property, and the amount of their taxes is called an **Assessment Roll.**

421. The officer who appraises the property and prepares the assessment roll is called an *Assessor.*

422. The **Quantities** to be considered are: 1. The *Taxable Property;* 2. The *Rate of Taxation;* 3. The *Amount of Tax.*

1. Real estate is often assessed by the proper officer for not more than one-half or two-thirds of its real value. The value of personal property may be given in by the owner under oath, or, if he neglects to do this, it may be valued by the officer.

2. After the taxes have been assessed, each person receives a notice of his taxation, stating the day of appeal, when he may appear before the proper officers and show reasons for correcting any mistakes that have been made.

WRITTEN EXERCISES.

1. The taxable property of a town is $576,000, and the rate of taxation $.006 on a dollar; what is the tax?

SOLUTION.—If the tax is $.006 on $1, on $576,000 it will be .006 times $576,000, or $3456.

OPERATION.
$576,000 × .006 = $3456

2. The real estate of a town is valued at $875,600, and the personal estate at $1,054,896; and there are 450 persons subject to a poll tax of $1.50 each; what is the whole tax, the rate being 6 mills on a dollar? *Ans.* $12,257.976.

423. **Table.**—In the assessment of taxes in a town, city, etc. a table is usually constructed by which the labor of calculation is greatly facilitated.

424. The following table is based on the rate of $.015 to the dollar. This table is used in Problems 3, 4, 5, 8:

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	.015	$10	.15	$100	$1.50	$1000	$15	$10,000	$150
2	.030	20	.30	200	3.00	2000	30	20,000	300
3	.045	30	.45	300	4.50	3000	45	30,000	450
4	.060	40	.60	400	6.00	4000	60	40,000	600
5	.075	50	.75	500	7.50	5000	75	50,000	750
6	.09	60	.90	600	9.00	6000	90	60,000	900
7	.105	70	1.05	700	10.50	7000	105	70,000	1050
8	.12	80	1.20	800	12.00	8000	120	80,000	1200
9	.135	90	1.35	900	13.50	9000	135	90,000	1350

3. Find by the table A's tax, whose property is $9540, and who pays a poll tax of $1.25.

SOLUTION.—We find from the table the tax on $9000 is $135, on $500 is $7.50, and on $40 is $.60; under these write $1.25, the poll tax; the sum will be the entire tax.

OPERATION.
Tax on $9000 = $135
" 500 = 7.50
" 40 = .60
" 1 poll = 1.25
Whole tax = $144.35

4. Find Mr. Wilson's tax, whose property is assessed at $7890, and who pays a poll tax of $1.25. *Ans.* $119.60.

5. My property is assessed at \$10,745, and my sister's at \$7536; what is the aggregate of our taxes? *Ans.* \$274.215.

6. What is the assessed value of property taxed \$87.50, at the rate of 5 mills on a dollar? *Ans.* \$17,500.

7. Mr. Nelson's entire tax is \$285; he pays a poll tax of \$1.25; the rate is 5 mills on the dollar; what is the valuation of his property? *Ans.* \$56,750.

8. Mr. Winthrop's property is assessed at \$17,000; he pays a poll tax of \$1.25 and .1½% on the income from his occupation, assessed at \$1000; what was his entire tax? *Ans.* \$257.75.

9. I have \$25,000 on interest, and my tax for money on interest is \$42, at 1½ mills on the dollar; for how much money at interest am I overtaxed? *Ans.* \$3000.

10. The expense of building a town-hall was \$10,750, which was raised by a tax on the property-holders of the town of 3¼ mills on the dollar; the collector's commission was 2¼%; what was the valuation of the property? *Ans.* \$3,383,828.447+.

NOTE.—The collector's commission is included in the tax.

11. I paid one year .35% township tax, .2½% county tax, .54% school tax, and \$1.50 poll tax; my whole tax was \$856.50; what was the value of my property? *Ans.* \$75,000.

12. Mr. Carter owns a house and lot worth \$45,000, the tax rate is 2¼%, and his tax bill \$675; what per cent. of the actual value of the property is the assessed value? *Ans.* 66⅔%.

13. A tax of \$14,250 is to be assessed on a town; the real estate is valued at \$1,200,000 and the personal property at \$750,000; there are 400 polls, each of which is taxed \$1.50; what is the rate of taxation? *Ans.* \$.007.

14. The trustees of a school expended for salary of teacher, \$600; fuel, \$54.25; apparatus, \$30.75. The school fund amounted to \$150, and the rest of the expenses was paid by a rate bill; if the entire attendance was 8320 days, what was A's bill, who sent three pupils 90 days each? *Ans.* \$17.36.

DUTIES OR CUSTOMS.

425. **Duties** are taxes levied by government upon imported goods. Duties are of two kinds, *ad valorem* and *specific.*

426. An **Ad Valorem** duty is a certain percentage assessed on the cost of the goods in the country from which they were imported.

427. A **Specific Duty** is a certain sum assessed on goods without regard to their cost.

428. **Tare** is an allowance for the weight of the box, cask, or covering containing the goods. Allowance is also made for *leakage* and *breakage.*

429. **Gross Weight** or **Value** is the weight or value of the goods before any deductions have been made.

430. **Net Weight** or **Value** is the weight or value of the goods after all allowances have been deducted.

431. An **Invoice,** or a *Manifest,* is an inventory or list of the goods and the prices at which they were purchased.

1. By the present tariff most duties of the United States are *ad valorem,* but some duties are specific, and some articles are charged both a specific and an *ad valorem* duty. The duty is reckoned on the actual cost at the place of purchase or manufacture, increased by all charges for transportation previous to final shipment.

2. All merchandise imported from foreign ports or places must be consigned in the manifest, invoice, or bill of lading to some person or firm at the port of importation, by whom it must be duly entered—either for immediate consumption or for warehouse.

3. Merchandise not intended for immediate consumption may be deposited in U. S. bonded warehouses, and remain there not longer than three years, the owner being at liberty to withdraw it at any time upon payment of the duties and charges for storage.

4. In custom-house business the *long* ton of 2240 lb., cwt. of 112 lb., and qr. of 28 lb. are used.

WRITTEN EXERCISES.

1. What is the specific duty, at $2\frac{1}{2}$¢ a lb., on 85 boxes of *raisins,* each containing 24 lb., tare $5\frac{1}{4}$ lb. a box?

SOLUTION.—We find the number of pounds to be 2040; multiplying 5¼ lb., the tare on 1 box, by 85, we have the whole tare, 446¼ lb., which, subtracted from 2040 lb., leaves the net weight, 1593¾ lb. The duty on 1594 lb. (calling the ¾ lb. 1 lb.) is $39.85.

OPERATION.
85 × 24 = 2040 lb.
5¼ × 85 = 446¼ lb. tare
2040 − 446¼ = 1593¾ lb.
1594 × 2½¢ = $39.85

Rules.—I. *For ad valorem duties, multiply the cost of the goods by the rate of duty.*

II. *For specific duties, deduct first the allowances, and compute the duty on the remainder.*

NOTE.—In reckoning duties, whole dollars, pounds, gallons, etc. are used as the base, fractions less than ½ being rejected, and more than ½ being reckoned as 1. Duties are payable in gold.

2. What is the duty on 375 yards of English worsted, invoiced at $2.25 a yard, at 20% ad valorem? *Ans.* $168.80.

3. I received from Havre an invoice of 80 dozen bottles of champagne, costing $10.50 per dozen; what is the duty at $6 a dozen, leakage 5%? *Ans.* $456.

4. Bailey & Co. received an invoice of Brussels laces costing 3600 francs in Brussels, charges 141.64 francs; what was the duty at 30%? *Ans.* $216.60.

5. What is the duty, at 35% ad valorem, on a bale of linen containing 56 webs, each 37½ yd. long, 30 inches wide, invoiced at 27¢ per square yard? *Ans.* $165.55.

6. Required the duty on 45 pieces of woolen bunting, each 38 yd. long, 42 in. wide, and costing 55¢ per yard, the rates of duty being 20¢ per square yard and 35% ad valorem? *Ans.* $728.35.

7. I imported 267 pieces of colored cotton goods, 30 inches wide, having 37 yards in each piece, @ 7¢; what will be the duty if there is a specific duty of 5½¢ a square yard and an ad valorem duty of 20%? *Ans.* $591.21½.

PROPERTY INSURANCE.

432. **Insurance** is a contract of indemnity for loss or damage within a given time. It is of two kinds: *Property Insurance and Personal Insurance.*

433. **Property Insurance** is security against loss by fire, transportation, etc. Insuring anything is called "taking a risk."

434. **Property Insurance** includes: *Fire Insurance, Marine Insurance, Transit Insurance*, and *Stock Insurance.*

1. *Transit Insurance* is security against loss by transportation by land or by both land and water; *Stock Insurance* is indemnity for loss of cattle, etc.

2. *Personal Insurance* includes *Life Insurance, Accident Insurance,* and *Health Insurance.*

3. *Accident Insurance* is indemnity for casualties to travellers and others; *Health Insurance* secures a weekly allowance in case of sickness.

435. **Fire Insurance** is security against loss by fire; *Marine Insurance* is security against loss by navigation.

436. The **Insurer**, or *Underwriter*, is the party or company taking the risk. The *Insured* or *Assured* is the party protected.

1. The *Policy* is the written agreement or contract between the insurers and the insured.

2. The *Premium* is the sum charged for insurance; it is a certain rate per cent. of the amount insured.

3. The *Sum Covered* by insurance is the amount insured on a property.

4. The *Base* is the amount insured on a property. The *Rate* varies with the risk.

The *Rate* of insurance is quoted at so many cents on the $100, or as so much per cent. Policies are renewed annually or at stated periods, and the premium is paid in advance. Risks are usually *rated* per annum. The rate for more than 1 year is determined by the following table:

The rate for 2 yr. is 1½ times the annual rate.
" " " 3 " " 2 " " " "
" " " 4 " " 2½ " " " "
" " " 5 " " 3 " " " "
" " " 7 " " 3½ " " " "

1. Insurance is generally done by ***stock companies.*** When an ***individual*** takes a risk, it is called an "out-door" business. A ***Mutual Insurance*** Company is one in which the profits and losses are shared by those who are insured.

2. To prevent fraud, companies will seldom insure the full value of property. In cases of loss the ***underwriters*** may either replace the property insured or pay its value. Only the amount of actual loss can be recovered, and often claims are ***adjusted*** for a part of the amount insured.

437. The **Quantities** considered are: 1. The *Amount Insured;* 2. The *Rate of Insurance;* 3. The *Premium;* 4. The *Valuation of the Property.*

438. Given the amount insured and the rate, to find the premium.

1. I insured my house for $4000 at 1¼%; required the amount of the premium.

OPERATION.

$4000
.01¼
$50.00

SOLUTION.—The premium on $4000 at 1¼% is .01¼ times $4000, which we find to be $50.

Principle.—*The premium equals the amount insured multiplied by the rate.*

1. To find the amount insured, ***divide the premium by the rate;*** to find the rate, ***divide the premium by the amount insured.***

2. To find what amount must be insured to cover the premium in case of loss, ***divide the valuation of the property by 1 minus the rate.***

WRITTEN EXERCISES.

2. Mr. Smith insured a factory valued at $10,500 for $7000 at 1¼%; what premium did he pay? *Ans.* $87.50.

3. I insured my house and furniture for $4500 for 5 years at 3¾%; required the premium. *Ans.* $168.75.

4. Professor Sanford insured his house for $5000, his library for $1500, and his furniture for $2000; what is the cost of insuring at 1½%, the policy costing $1.50? *Ans.* $129.

5. The premium paid for insuring ⅔ of the value of a house for 3 years, at 2½%, was $100; what was the value of the house? *Ans.* $6000.

6. I insured my house, worth $8000, for ¾ of its value at 2¼%, for 5 years, so as to include the premium if burned; required the amount insured. *Ans.* $6138.11.

7. Mr. Stone effected an insurance of $12,000 on a hotel, paying $91.25, including $1.25, the cost of the policy; what was the rate of insurance? *Ans.* ¾%.

8. Insured my house for $6000 for 3 years at an annual rate of ⅞%; what premium did I pay? *Ans.* $105.

NOTE.—For 3 years the premium would be 2 times the annual rate. See table under Art. 436.

9. Copeland & Co. paid $196.87½ for a 7-year policy on their stable, the annual rate being ¾%; what is the amount of the policy? *Ans.* $7500.

10. I insured $15,000 on my factory and $10,000 on the machinery for a period of 5 years, paying $658.75, including $2.50, the cost of the policy; what would the premium be rated at annually? *Ans.* ⅞%.

11. Bingham & Hollis insured a vessel in the Ætna Insurance Co. against fire for $80,000, at ¾%, and also had a marine insurance of ¼ as much on the cargo, at 1⅜%; ½ of the cargo was thrown overboard in a storm, and the vessel on her way home was destroyed by fire; what was the loss to the underwriters? *Ans.* $89,125.

12. A store in Chicago valued at $50,000, and a stock of goods worth $75,000, were insured for ¾ of their value at 1¼%; what was the loss of the owners, and what the loss to the underwriters, if the store and goods were destroyed in the great fire of 1872? *Ans.* $32,421.87½; $92,578.12½.

LIFE INSURANCE.

439. **Life Insurance** is a contract by which a company agrees to pay a certain sum of money to a person when he attains a certain age, or to his heirs at his death.

440. Life insurance companies are either **Stock** or **Mutual.**

1. In *Stock Companies* the capital is subscribed as in any other corporation, and all profits or losses are divided among the stockholders.

2. In *Mutual Companies* the profits or losses are divided among the insured. The profits may go to decrease the premiums or increase the policies.

3. In *Assessment Companies* no regular premium is paid, but whenever a loss occurs each person insured is assessed for his share of the loss.

441. The **Policies** of Life Insurance most frequently used are the following:

1. *Life Policies,* payable at the death of the insured, premium payable annually during life or in one, five, or ten annual payments.

2. *Endowment Policies,* payable to the insured at the end of a certain number of years, or to his heirs if he dies sooner, premium payable either annually during the continuance of the policy or in one or more annual payments.

NOTE.—Insurance companies use tables in their calculations showing the amount of premium that must be paid on $1000 at different ages. In the following examples the rates are taken directly from these tables.

WRITTEN EXERCISES.

1. At 40 years of age, what must I pay annually for a life policy of $5000, the premium being $31.50 per $1000?

SOLUTION.—The premium on $1000 is $31.50; hence, on $5000 it will be 5.000 times $31.50, which is $157.50.

OPERATION.

$\$31.50 \times 5.000 = \$157.50$

Principle.—*The premium equals the premium on* $1000 *multiplied by the amount of the policy divided by* 1000.

2. What is the annual premium on a life policy of $6000, the rate being $33.90 on $1000? *Ans.* $203.40.

3. Mr. Jones insured his life for $15,000, at an annual cost of $397.50; what was the rate on $1000? *Ans.* $26.50.

4. A lady insures her life for $8000, at an annual payment of $29.30 per $1000; if she lives 15 years, what amount will she have paid in premiums? *Ans.* $3516.

5. If a man whose life is insured for $20,000, at an annual premium of $24.80 per $1000, dies after making 8 payments, how much more than the amount of the premium will be paid to his heirs? *Ans.* $16,032.

6. Mr. Cornwell took out a policy in the N. E. Mutual Life Ins. Co. for $30,000, paying a premium of $22.70 per $1000; he made 20 payments, and 12% of the premium was returned as dividends; how much did the insurance actually cost him? *Ans.* $11,985.60.

7. Mr. Westlake took out an endowment policy for $10,000, payable in 20 years, the premium being $47.45 per $1000; if he lives to receive the endowment, will he have paid more or less than if he had taken a policy of the same amount 5 years later for 15 years at $67.15 per $1000? *Ans.* $582.50 less.

REVIEW PROBLEMS.

1. The list price of a lot of ginghams is 20¢ a yard; if I buy them at 20% off and sell them at 25% on, what is my gain per yard? *Ans.* 9¢.

2. Mr. Watson bought mining stock ($50) at 9% premium, and sold it at a loss of $5 on a share; at what rate was it sold? *Ans.* 1% disc.

3. Bought French merinos at $1.15 a yard, and marked them *cln*, my key being "Charleston;" what was my gain % if I sold the goods at the marked price? *Ans.* $30\frac{10}{23}$%.

4. An agent sold 150 barrels of flour, charging $2\frac{1}{2}$% commission and $2\frac{1}{4}$% guaranty; the net proceeds due the consignor were $1143; for how much was the flour sold per barrel? *Ans.* $8.

SIMPLE INTEREST.

442. **Interest** is money charged for the use of money for a certain time.

443. The **Principal** is the sum for which interest is charged. Interest is reckoned as a percentage of the principal.

444. The **Rate** of interest is the interest on $1 for a certain time. The usual time is one year.

445. The **Time** is the period during which the money is on interest.

446. The **Amount** is the sum of the principal and interest.

447. **Simple Interest** is interest on the principal only. **Compound Interest** is interest also on the interest.

448. **Legal Interest** is interest at the rate fixed by law. It varied in different States in 1895 as follows:

STATES.	RATE %		STATES.	RATE %		STATES.	RATE %	
Alabama . .	8	8	Kentucky . . .	6	6	North Dakota.	7	12
Alaska . . .	8	10	Louisiana . . .	5	8	Ohio	6	8
Arizona. . .	7	*	Maine	6	*	Oklahoma . .	7	12
Arkansas . .	6	10	Maryland . . .	6	6	Oregon . . .	8	10
California. .	7	*	Massachusetts .	6	*	Pennsylvania.	6	6
Colorado . .	8	*	Michigan . . .	6	8	Rhode Island.	6	*
Connecticut .	6	6	Minnesota. . .	7	10	South Carolina	7	8
Delaware . .	6	6	Mississippi . .	6	10	South Dakota.	7	12
Dist. Columbia	6	10	Missouri . . .	6	8	Tennessee . .	6	6
Florida . . .	8	10	Montana . . .	10	*	Texas	6	10
Georgia. . .	7	8	Nebraska . . .	7	10	Utah	8	*
Idaho . . .	10	18	Nevada. . . .	7	*	Vermont. . .	6	6
Illinois . . .	5	7	New Hampshire	6	6	Virginia. . .	6	6
Indiana. . .	6	8	New Jersey . .	6	6	Washington .	8	*
Indian Ter. .	6	10	New Mexico. .	6	12	West Virginia	6	6
Iowa	6	8	New York. . .	6	6	Wisconsin . .	6	10
Kansas . . .	6	10	North Carolina	6	8	Wyoming . .	12	*

The first column gives the legal rate; second column, the rate that may be agreed upon; the * indicates no limit to the rate.

449. **Usury** is a rate of interest greater than the law allows. Various penalties are attached to taking usury.

450. The **Quantities** are five: 1. The *Principal;* 2. The *Interest;* 3. The *Rate;* 4. The *Time;* 5. The *Amount.*

1. In notes, contracts, accounts, mortgages, etc., when no rate is specified, the legal rate is understood.

2. In computing interest it is customary to reckon a month as $\frac{1}{12}$ of a year, and a day as $\frac{1}{30}$ of a month. In dealing with the U. S. Government each day is $\frac{1}{365}$ of a year.

THE METHOD FOR YEARS.

ORAL EXERCISES.

1. What is the interest of $60 for 2 yr. 6 mo. at 6%?

SOLUTION.—6 months equal $\frac{6}{12}$, or $\frac{1}{2}$ of a year, which with 2 yr. equals $2\frac{1}{2}$, or $\frac{5}{2}$ years. At 6 per cent. for 1 yr., $\frac{6}{100}$ of the principal equals the interest, and for $2\frac{1}{2}$, or $\frac{5}{2}$ yr., $\frac{5}{2}$ times $\frac{6}{100}$, or $\frac{15}{100}$ or $\frac{3}{20}$ of the principal, equals the interest; $\frac{3}{20}$ of $60 equals $9.

What is the interest of

2. $40 for 5 yr. at 5%?

3. $60 for 5 yr. at 4%?

4. $80 for 4 yr. at 5%?

5. $600 for 2 yr. 3 mo. at 8%?

6. $250 for 4 yr. 8 mo. at 6%?

7. $220 for 3 yr. 9 mo. at 8%?

8. $330 for 7 yr. 6 mo. at 4%?

9. $400 for 3 yr. 7 mo. 6 da. at 5%?

10. What is the amount of $50 for 3 yr. 6 mo. at 8 per cent.?

REMARK.—We find that $\frac{28}{100}$, or $\frac{7}{25}$ of the principal, equals the interest; hence, $\frac{32}{25}$ of the principal equals the amount; $\frac{32}{25}$ of $50 equals $64.

11. What is the amount of $300 for 7 yr. 10 mo. at 6 per cent.?

12. What is the amount of $400 for 5 yr. 4 mo. at 6 per cent.?

WRITTEN EXERCISES.

1. What is the interest of $3600 for 4 yr. 7 mo. 15 da. at 6%?

SOLUTION.—By reduction, we find that 4 yr. 7 mo. 15 da. equals $4\frac{5}{8}$ yr. At 6%, the interest for 1 year equals .06 times $3600, which is $216; if the interest for 1 year is $216, the interest for $4\frac{5}{8}$ yr. is $4\frac{5}{8}$ times $216, which, by multiplying, we find is $999. Hence the following

OPERATION.

$$\begin{array}{r} \$3600 \\ \underline{.06} \\ \$216.00 \\ \underline{4\frac{5}{8}} \\ \$999.00 \end{array}$$

Rule.—I. *To find the interest, multiply the principal by the rate, and that product by the time expressed in years.*

II. *To find the amount, add the interest to the principal.*

Required the interest

2. Of \$250 for 4 yr. 3 mo. at 6%. *Ans.* \$63.75.
3. Of \$680 for 5 yr. 4 mo. at 8%. *Ans.* $\$290.13\frac{1}{3}$.
4. Of \$860 for 5 yr. 9 mo. at 5%. *Ans.* \$247.25.
5. Of \$960 for 8 yr. 5 mo. at 7%. *Ans.* \$565.60.
6. Of \$590 for 3 yr. 10 mo. at 8%. *Ans.* $\$180.93\frac{1}{3}$.
7. Of \$1089 for 4 yr. 7 mo. 6 da. at 4%. *Ans.* \$200.376.
8. Of \$1040 for 3 yr. 3 mo. 9 da. at 5%. *Ans.* \$170.30.

THE SIX PER CENT. METHOD.

451. The **Six Per Cent. Method** is so called because the process is based upon that rate.

1. What is the interest of \$480 for 8 yr. 10 mo. 18 da. at 6%?

SOLUTION.—The Int. of \$1 for 1 yr. is \$0.06, and for 8 yr. it is 8 times \$0.06, or \$0.48.

The Int. of \$1 for 1 mo., or $\frac{1}{12}$ of a year, is $\frac{1}{12}$ of 6¢, or $\frac{1}{2}$ of a cent, and the Int. for 10 mo. is 10 times $\frac{1}{2}$ of a cent, or \$0.05.

The Int. of \$1 for 1 mo., or 30 da., is $\frac{1}{2}$ of a cent, or 5 mills, and for 1 day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill, and for 18 days it is 18 times $\frac{1}{6}$ of a mill, or \$0.003.

OPERATION.

$$\begin{array}{r} 8\times .06 = \$0.48 \\ \frac{1}{2}\times 10 = \ \ .05 \\ \frac{1}{6}\times 18 = \ \ .003 \\ \hline \$0.533 \\ 480 \\ \hline \$255.84 \end{array}$$

Adding these results, we have \$0.533 as the Int. of \$1 for 8 yr. 10 mo. and 18 da., and on \$480 it is 480 times \$0.533, or \$255.84.

Rule.—I. *Multiply the rate,* .06, *by the number of years; take* $\frac{1}{2}$ *of the number of months as cents, and* $\frac{1}{6}$ *of the number of days as mills; their sum will be the interest of* \$1 *for the given time at* 6%.

II. *Multiply this sum by the principal, and the product will be the interest of the principal at* 6%. *For any other rate, take as many sixths of this interest as the rate is of six.*

1. Another method is to reduce the years to months, and take *half* the number of *months* for *cents*, etc., as before.

2. Another method is to take the *number* of *months* as *cents*, and *one-third* of the number of *days* as *mills*, and multiply the sum by *half the principal.*

3. The method for days, popularly expressed, is, "*Multiply dollars by days, and divide by* 6000."

WRITTEN EXERCISES.

Required the interest of

2. \$450 for 4 yr. 8 mo. 12 da. at 6%. *Ans.* \$126.90.

3. \$996 for 6 yr. 4 mo. 15 da. at 6%. *Ans.* \$380.97.

4. \$765 for 8 yr. 7 mo. 6 da. at 6%. *Ans.* \$394.74.

5. \$84.75 for 8 yr. 9 mo. 24 da. at 7%. *Ans.* \$52.30+.

6. \$23.75 for 7 yr. 7 mo. 21 da. at 5%. *Ans.* \$9.07+.

7. \$.475 for 11 yr. 5 mo. 14 da. at 8%. *Ans.* $\$.43\frac{1}{2}$+

8. $\$147.37\frac{1}{2}$ for 4 yr. 11 mo. 13 da. at 7%. *Ans.* \$51.094

9. $\$752.87\frac{1}{2}$ for 9 yr. 8 mo. 10 da. at 9%. *Ans.* \$656.88+

10. $\$387.18\frac{3}{4}$ for 10 yr. 7 mo. 7 da. at 10%. *Ans.* \$410.53– ,

11. \$850.10 for 5 yr. 5 mo. 5 da. at $6\frac{3}{4}$%. *Ans.* \$311.61+ .

12. $\$890.83\frac{1}{3}$ for 8 yr. 9 mo. 17 da. at $7\frac{1}{2}$%. *Ans.* \$587.76+.

THE 60-DAY METHOD.

452. At 6% a year the rate for 2 mo., or 60 da., is 1%; hence, *for* 60 *da.*, $\frac{1}{100}$ *of the principal equals the interest.* From this we have the following method, called *the 60-Day Method.*

Rule.—*Point off two places in the principal for the interest for* 60 *days, and take multiples or aliquot parts of this interest for any other number of days.*

1. What is the interest of \$240 at 6% for 66 da.? for 96 da.?

SOLUTION.—Pointing off two places, we have \$2.40, the Int. for 60 da.; then take $\frac{1}{10}$ of \$2.40, we have \$0.24, the Int. for 6 da.; then the sum of these interests, or \$2.64, is the interest for 66 da.

OPERATION.

\$2.40 = Int. for 60 da.
.24 = " " 6 da.
\$2.64 = " " 66 da.

SOLUTION.—Pointing off two places, we have \$2.40, the Int. for 60 da.; take $\frac{1}{2}$ of \$2.40 for the Int. of 30 da., and $\frac{1}{10}$ of \$2.40 for the Int. for 6 da.; their sum is the Int. for 96 da.

OPERATION.

\$2.40 = Int. for 60 da.
1.20 = " " 30 da.
.24 = " " 6 da.
\$3.84 = " " 96 da.

2. Find the Int. of $720, at 6%, for 6 mo. 12 da. For 128 da.

OPERATION.

$7	20 = Int. for 60 da.
21	60 = " " 6 mo. (3×2 mo.).
1	20 = " " 10 da. ($\frac{1}{6}$ of 60 da.).
	24 = " " 2 da. ($\frac{1}{5}$ of 10 da.).
$23.04 = Int. for 6 mo. 12 da.	

OPERATION.

$7	20 = Int. for 60 da.
14	40 = " " 120 da.
	72 = " " 6 da.
	24 = " " 2 da.
$15.36 = " " 128 da.	

WRITTEN EXERCISES.

Required the interest

3. Of $1800 for 6 mo. at 6%. *Ans.* $54.

4. Of $450 for 9 mo. at 6%. *Ans.* $20.25.

5. Of $750 for 3 mo. 10 da. at 6%. *Ans.* $12.50.

6. Of $1260 for 33 da. at 6%. *Ans.* $6.93.

7. Of $760 for 63 da. at 6%. *Ans.* $7.98.

8. Of $150 for 135 da. at 6%. *Ans.* $3.37½.

9. Of $720 for 5 mo. 22 da. at 6%. *Ans.* $20.64.

10. Of $1250 for 4 mo. 29 da. at 7%. *Ans.* $36.21+.

11. Of $325 for 1 yr. 7 mo. 12 da. at 7%. *Ans.* $36.78—.

12. Of $875 for 2 yr. 3 mo. 24 da. at 8%. *Ans.* $162.17—.

13. Of $621.50 for 3 yr. 7 mo. 15 da. at 5%. *Ans.* $112.65—.

14. Of $360.50 for 5 yr. 4 mo. 16 da. at 4½%. *Ans.* $87.24+.

15. On a 3 mo. note for $2400, dated March 21, 1894, at 6%, reckoning exact time.

EXACT TIME.

In March,	10 da.
In April,	30 da.
In May,	31 da.
In June,	21 da.
	92 da.

OPERATION.

$24	00 = Int. for 60 da.
12	00 = " " 30 da.
	80 = " " 2 da.
$36.80	

16. On a 3 mo. note for $4500, dated Feb. 20, 1896, at 6%, reckoning exact time. *Ans.* $67.50.

17. Of $950, from April 14 to Sept. 21, at 6%, exact number of days. *Ans.* $25.33⅓.

18. Of $480, from June 16 to Nov. 10, at 7%, exact number of days. *Ans.* $13.72.

19. On a note for $640, dated Oct. 9, 1894, and due Jan. 15, 1895, at 6%, allowing 3 days of grace. *Ans.* $$10.77\frac{1}{3}$.

20. On a note for $1250, dated Nov. 16, 1894, and due Mar. 25, 1895, at 6%, allowing 3 days of grace. *Ans.* $27.50.

NOTE.—For Method by Cancellation, see Supplement.

METHOD OF EXACT INTEREST.

453. **Exact Interest** is interest obtained by reckoning 365 days to the year.

454. **Exact Interest** is reckoned by the United States Government, and is growing in favor with business-men.

455. Bankers and business-men often use *Interest Tables,* which are sometimes calculated to exact interest.

1. What is the exact interest of $875 from October 15 to February 12 at 7%?

SOLUTION.—From Oct. 15 to Feb. 12 there are 120 days; the interest of $875 for 1 year of 365 days, at 7%, is $61.25, and for 120 days it is $\frac{120}{365}$, or $\frac{24}{73}$ of $61.25, which is $20.14–.

OPERATION.

```
      $875
       .07
    $61.25
        24
73)1470.00
   20.13 51/73
```

Rule.—*Find the interest for the integral number of years; then multiply the interest for one year by the exact number of days and divide by* 365; *the sum of the two results will be the exact interest.*

NOTE.—The exact interest for any number of days less than one year may also be found by deducting from the common interest $\frac{1}{73}$ of itself.

WRITTEN EXERCISES.

2. What is the interest, at 4%, of $484.50, from Jan. 15th 1894, to July 18th, 1896? *Ans.* $48.58.

3. What is the amount of $540, on interest at 6%, from June 12th, 1887, to Oct. 30th, 1896? *Ans.* $844.03–.

4. A had $1800 on interest from May 20th, 1892, to Sept. 5th, 1897; what was the interest at $5\frac{1}{2}\%$? *Ans.* $524.29+.

5. Required the amount of $1500.75, on interest at 5% from June 29th, 1890, to Jan. 11th, 1900. *Ans.* $2216.38+.

6. Prove that $\frac{1}{73}$ deducted from common interest will give exact interest.

INTEREST ON PROMISSORY NOTES.

456. A **Promissory Note** is a written promise to pay a certain sum of money on demand or at a specified time.

457. The **Maker** of a note is the party who signs it.

458. The **Payee** is the party to whom it is made payable. The *Holder* is the one who owns it.

459. The **Face** of a note is the sum whose payment is promised.

The face is written in words in the body of the note to avoid error or fraud. The cents are usually written in figures as hundredths of a dollar.

FORMS OF NOTES.

$450.75. NEW YORK, *Aug.* 10, 1895.

Two months after date, I promise to pay Mary Martin, or order, Four Hundred Fifty $\frac{75}{100}$ *Dollars, for value received.*

JAMES JOHNSON.

$468.25. PHILADELPHIA, *Dec.* 21, 1894.

For value received, three months after date, I promise to pay Joseph Stanton, or order, Four Hundred Sixty-eight $\frac{25}{100}$ *Dollars, without defalcation.* THOMAS RICHARDS.

460. In the above promissory notes let the pupil point out the *maker*, the *payee*, the *face*, the *date*, etc.

1. A note should contain the words "value received," otherwise the holder may be required to prove that value was received. In business language a note is said to be "made in favor of" the payee.

2. If a note reads "with interest," it draws interest from date; otherwise it draws interest from the time of maturity until paid. A note

may draw interest from a particular time after date if so specified in the note.

461. A **Negotiable Note** is a note that can be transferred from one party to another. A note not thus transferable is called a *Non-negotiable Note.*

1. A note is negotiable when it is made payable to the "bearer" or to the "order" of the payee. In order to transfer it to a second party the payee writes his name on the back of it, which is called *indorsing* the note. The person who writes his name on the back of a note as security for its payment is called the *Indorser.*

2. The words "without defalcation" are required in Pennsylvania to make a note negotiable; in New Jersey, "without defalcation or discount;" in Missouri, "negotiable and payable without defalcation or discount."

462. In many of the States a note *matures*, or is legally due, *three days* after the time specified in the note. These three days are called *Days of Grace.*

1. In several of the States, as New York, Pennsylvania, New Jersey, Illinois, etc., days of grace are abolished by law.

2. If a note falls due on Sunday or a legal holiday, it is to be paid in most States on the business-day preceding. In Pennsylvania it is to be paid the next business-day following.

3. When the time of a note is stated in months, calendar months are meant. A 4 mo. note, dated Oct. 15, would mature Feb. 15; but if dated Oct. 29, 30, or 31, it would mature on the last day of February.

463. A **Protest** is a written notice by a *notary public* to the indorsers that the note has not been paid.

1. A protest must be made out on the day the note matures, and be sent to the indorser immediately, to *hold him responsible.* In Pennsylvania a note is not to be protested on Saturday, but on the next business-day following.

2. The neglect to protest a note on maturity releases an indorser from all obligation to pay it unless the words "waiving demand and notice" appear above the indorser's signature.

464. The **Principal Kinds** of notes are the *Time Note, Joint Note*, and *Joint and Several Note.*

1. A *Time Note* is one made payable at a specified time; when no time of payment is specified, the note is due on *demand.* A *Joint Note* is a note signed by two or more persons, who are jointly liable for its

payment. A *Joint* and *Several Note* is a note signed by several persons, who are both jointly and singly liable for its payment.

2. In raising money on notes it is customary to have one or more responsible persons write their names on the back of the note as security for its payment. In case of the refusal of the maker to pay the note when due, each indorser is liable for the whole amount of the note in the order of signing, unless he writes above his name the words "without recourse," or unless there is an agreement between two or more indorsers to share the loss between them.

3. When the maker fails to pay a note, it is usual for the holder to make his demand on the last *liable* indorser, who pays the note and then gets the amount from the preceding indorser, and so on, up to the first indorser. The holder, however, has the option of collecting the amount from *any* liable indorser, and when so collected all *subsequent* indorsers are released, the indorser who pays becomes the holder, and may collect from any *prior* liable indorser, and so on up to the first.

WRITTEN EXERCISES.

1. TIME NOTE.

$375. PHILADELPHIA, *Nov.* 18, 1894.

Three months after date, I promise to pay William Stevens, or order, Three Hundred Seventy-five Dollars, with interest, for value received, without defalcation. EDWARD CARTER.

What will be due on this note at maturity?

EXACT TIME.	
Nov.	12 da.
Dec.	31 da.
Jan.	31 da.
Feb.	18 da.
Time,	92 da.

OPERATION.

3 | 75 = Int. for 60 da.
1 | 875 = " " 30 da.
| 125 = " " 2 da.
5.750
375
$380.75, Amount due.

2. JOINT NOTE.

$800. NEW YORK, *Jan.* 7, 1895.

For value received, we promise to pay Philip Garret, or order, on demand, Eight Hundred Dollars, with interest, at 6 per cent.

THOMAS CHURCHILL,
CHARLES CHASE.

What will be due on this note July 1, 1895? *Ans.* $823.33.

3. JOINT AND SEVERAL NOTE.

$640.75. ST. LOUIS, MO., *Feb.* 21, 1895.

Four months after date, we jointly and severally promise to pay George F. Bissell, or order, Six Hundred Forty $\frac{75}{100}$ *Dollars, with interest, for value received, negotiable and payable without defalcation or discount.*

GEORGE ELLIS,
EDWARD PRESTON.

What will be due on this note at maturity? *Ans.* $653.56.

4. COMPANY NOTE PAYABLE AT A BANK.

$560. NEW YORK, *March* 17, 1895.

Sixty days after date, for value received, we promise to pay James Ferris, or order, at the Chemical Bank, Five Hundred Sixty Dollars, with interest.

FUNK, WAGNALLS & CO.

What will be due on this note at maturity? *Ans.* $565.60.

5. INDIVIDUAL NOTE PAYABLE AT A BANK.

$347.25. NEWARK, N. J., *Dec.* 29, 1895.

Six months after date, I promise to pay John Morris, or order, at the Third National Bank, Three Hundred Forty-seven $\frac{25}{100}$ *Dollars, with interest, for value received, without defalcation or discount.*

GEORGE ANDREWS.

What will be due on this note at maturity? *Ans.* $357.84.

WRITING NOTES.

6. Write a negotiable note for $650.28, making yourself the payee and E. M. Hayward the maker; legal rate of interest.

7. Write a non-negotiable note for $412.65, making Charles Morton the payee and yourself the maker, payable on demand, with legal interest.

8. Write a note from the following data: Face, $500; negotiable; time, ninety days; payee, Alfred Foster; maker, Howard Truman; rate of interest, 6%; place, Lancaster, Pa.

PARTIAL PAYMENTS.

465. **Partial Payments** are payments in part of a note or other obligation.

The amounts and dates of the payments are usually written upon the back of the note or obligation. The person holding the obligation signs his name to this statement as a receipt.

466. An acknowledgment of a payment written on the back of the note or obligation, stating the amount and date of the payment, is called an *Indorsement.*

The term *Indorsement* is used in different business papers; in each case, however, it means a *writing on the back,* from the Latin *dorsum,* the back.

1. The writing of the name on the back of a check, draft, note, etc. is called a *General Indorsement,* or an *indorsement in blank.*

2. A *Special Indorsement* directs the obligation to be paid to some particular person or to his order.

467. The Supreme Court of the United States, and nearly all the States, adopt the following rule for partial payments, called

THE UNITED STATES RULE.

I. *Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest, subtract the payment from the amount and treat the remainder as a new principal.*

II. *If the payment is less than the interest, find the amount of the same principal to the time when the sum of the payments shall equal or exceed the interest due, and subtract the sum of the payments from the amount.*

III. *Proceed in the same manner with the remaining payments until the time of settlement.*

1. This rule is founded upon the decision of Chancellor Kent. The principle is, that neither interest nor payment shall draw interest. It has been adopted by nearly all the States—New Hampshire, Vermont, and Connecticut being the principal exceptions.

2. The time between the dates is generally found by compound subtraction.

1. $500. NEWARK, N. J., *April* 10, 1893.

Two years after date, for value received, I promise to pay James Hill, or order, Five Hundred Dollars, with interest, without defalcation or discount. WILLIAM NEWHALL.

Indorsements: Jan. 1, 1894, $100; Nov. 15, 1894, $150.

How much was due April 10, 1895?

OPERATION.

Principal	$500.00
Interest to Jan. 1, 1894, 8 mo. 21 da.	21.75
Amount	521.75
First payment to be deducted	100.00
Remainder for new principal	421.75
Interest to Nov. 15, 1894, 10 mo. 14 da.	22.072
Amount	443.822
Second payment to be deducted	150.000
Remainder for new principal	293.822
Interest to April 10, 1895, 4 mo. 25 da.	7.101
Amount due at time of settlement	$300.923

2. $750. PHILADELPHIA, *May* 10, 1892.

Three years after date, for value received, I promise to pay Thomas Ellis, or order, Seven Hundred Fifty dollars, with interest, without defalcation. SAMUEL MIDDLETON.

Indorsements: Jan. 15, 1893, $124.75; Sept. 12, 1893, $20; Dec. 16, 1894, $216.80. How much remained due May 10, 1895?

OPERATION.

Principal		$750.000
Interest to Jan. 15, 1893, 8 mo. 5 da.		30.625
Amount		780.625
First payment to be deducted		124.75
Remainder for new principal		655.875
Interest to Sept. 12, 1893, 7 mo. 27 da.		25.907
Second payment less than interest due	$20.00	
Int. on $655.875 to Dec. 16, 1894, 1 yr. 3 mo. 4 da.		49.628
Amount		731.410
Third payment to be added to second	$216.80	236.800
Remainder for new principal		494.610
Interest to May 10, 1895, 4 mo. 24 da.		11.870
Amount due on settlement		$506.48

NOTE.—The interest may be calculated from the time of the first payment to that of the third, if preferred.

3. $\overline{\$600.}$ CHICAGO, *Jan.* 11, 1892.

For value received, on demand, I promise to pay Philip Butler, or order, Six Hundred Dollars, with interest.

CHARLES MORTON.

Indorsements: Dec. 18, 1892, \$60; March 31, 1893, \$85; Sept. 21, 1893, \$67.20.

The note was paid March 1, 1894; what was then due?
Ans. \$445.29.

NOTE.—The legal rate in Illinois is 5%; see Table, p. 283.

4. $\overline{\$3000.}$ INDIANAPOLIS, IND., *Oct.* 20, 1890.

Three months after date, I promise to pay William Dennison, or order, Three Thousand Dollars, for value received, with interest from date. MORRIS PRITCHARD.

Indorsements: June 15, 1892, \$500; Aug. 18, 1893; \$750; Sept. 9, 1894, \$400.

How much was due Oct. 20, 1895? *Ans.* \$2120.08.

5. A note for \$4500 was given May 1, 1893, and was indorsed as follows: Oct. 17, 1893, \$250; Feb. 18, 1894, \$345; July 10, 1894, \$217.50; what was due May 1, 1895, interest at 7%? *Ans.* \$4280.06.

6. A note for \$7460 was given June 11, 1891; indorsed Dec. 8, 1891, \$200; March 25, 1893, \$500; June 15, 1894, \$225; Sept. 7, 1894, \$150; what was due at settlement, Jan. 1, 1895, interest 4%? *Ans.* \$7431.27.

MERCANTILE RULE.

468. Merchants often find the balance due on notes, payable within a year, on which partial payments have been made, by the following rule:

I. *Find the amount of the principal to the time of settlement, and also the amount of each payment to the time of settlement.*

II. *To find the balance due, subtract the sum of the amounts of the payments from the amount of the principal.*

1. As the periods in these notes are all short, the interest should be calculated for the exact number of days.

2. The method of EXACT INTEREST is sometimes used, but in the following problems 360 days are reckoned to a year.

WRITTEN EXERCISES.

1. $8500. HARRISBURG, *June* 18, 1893.

Thirty days after date, for value received, I promise to pay Henry Landis, or order, Eight Thousand Five Hundred Dollars, without defalcation. ANDREW HERR.

Indorsements: July 30, $150; Sept. 12, $300; Oct. 9, $500; Dec. 15, $700.

What is due June 15, 1894?

OPERATION.

Amount of

$8500 from	July 18, 1893,	to June 15, 1894,	332 da.		$8970.33
150 "	" 30,	" " " "	320 da.	$158.00	
300 "	Sept. 12,	" " " "	276 da.	313.80	
500 "	Oct. 9,	" " " "	249 da.	520.75	
700 "	Dec. 15,	" " " "	182 da.	721.23	1713.78
	Balance due June 15, 1894,				$7256.55

NOTE.—This note does not begin to draw interest till July 18.

2. A note for $3500, given Oct. 9, 1892, has the following indorsements: Jan. 1, 1893, $200; Feb. 21, $300; April 17, $400; June 16, $350; Sept. 12, $500; what was due Oct. 1, 1893, at 5%? *Ans.* $1890.91.

3. A note for $4564.25, dated April 16, 1891, has the following indorsements: June 1, $345; Aug. 20, $742.50; Oct. 15, $324.25; Dec. 10, $600; Jan. 15, 1892, $560; what was due April 16, 1892? *Ans.* $2186.92.

4. A note for $6000, at 60 days, dated Aug. 26, 1894, has the following indorsements: Oct. 31, $200; Jan. 18, 1895, $400; April 20, 1895, $500; July 10, $150; what was due Aug. 1, 1895? *Ans.* $4998.73.

NOTES.—1. Notice in Ex. 3 that 1892 is a leap year.

2. For Connecticut, Vermont, and New Hampshire rules, see *Brooks's Higher Arithmetic.*

PROBLEMS IN SIMPLE INTEREST.

469. In **Simple Interest** there are three cases besides the finding of the interest on a sum of money. These will now be considered.

470. Given the time, the rate, and the interest or the amount, to find the principal.

ORAL EXERCISES.

1. What principal will in 4 yr. 2 mo., at 6%, give $70 interest?

SOLUTION.—We find that $\frac{1}{4}$ of the principal equals the interest, which is $70; if $\frac{1}{4}$ of the principal is $70, $\frac{4}{4}$, or the principal, equals 4 times $70, or $280.

What principal will give an interest of

2. $72 in 6 yr. at 4%?

3. $70 in 7 yr. at 5%?

4. $27 in 9 yr. at 6%?

5. $21 in 7 yr. 6 mo. at 7%?

6. $54 in 2 yr. 3 mo. at 8%?

7. $53 in 8 yr. 10 mo. at 6%?

8. How much money must a person borrow at 6% that the annual interest shall be $120?

9. How much money has William on interest, supposing he receives $480 for 5 yr. 4 mo. at 6%?

WRITTEN EXERCISES.

1. What principal will in 3 yr. 6 mo., at 6%, give $136.50 interest?

SOLUTION.—We find the interest of $1 for 3 yr. 6 mo., at 6%, is $.21. If $1 gives an interest of $.21, to give $136.50 interest it will require as many dollars as $.21 are contained times in $136.50, which is $650. Hence the following

OPERATION.

3 yr. 6 mo. = 42 mo.

$42 \div 2 = .21$

$$\frac{\$136.50}{.21} = \$650$$

Rule.—*Divide the given interest by the interest of* $1 *for the given rate and time, or divide the amount by the amount of* $1.

What principal will

2. Give $476 interest in 4 yr. 8 mo. at 6%? *Ans.* $1700.

3. Give $72.80 interest in 10 yr. 10 mo. at 7%? *Ans.* $96.

4. Amount to \$1200.50 in 8 yr. 4 mo. at 8%? *Ans.* \$720.30.

5. Give \$575.40 in 5 yr. 8 mo. 15 da. at 5%? *Ans.* \$2016.

6. Am't to \$2299.75 in 7 yr. 7 mo. 12 da. at 7%? *Ans.* \$1500.

7. Give \$988.40 in 5 yr. 10 mo. 18 da. at 7%? *Ans.* \$2400.

8. The sum of A's and B's money on interest for 3 yr. 7 mo. at 6%, gives \$3440 interest; how much money has each, if 3 times B's equals A's? *Ans.* \$12000; \$4000.

471. Given the principal, the rate, and the interest or the amount, to find the time.

ORAL EXERCISES.

1. In what time will \$400 at 6% give \$144 interest?

SOLUTION.—At 6 per cent. for *one* year, $\frac{3}{50}$ of the principal equals the interest; $\frac{3}{50}$ of \$400 is \$24; if it require *one* year for \$400 to gain \$24, to gain \$144 it will require as many years as \$24 are contained times in \$144, which are 6 years.

In what time will

2. \$200 at 4% give \$48 int.?
3. \$150 at 5% give \$30 int.?
4. \$300 at 8% give \$72 int.?
5. \$60 at 7% give \$21 int.?
6. \$75 at 6% am't to \$111?
7. \$50 at 9% am't to \$86?

8. In what time will a principal at 10% gain 2 times itself? 3 times itself? 4 times itself?

9. In what time will a principal double itself at 5%? at 6%? at 8%? at 12½%? Treble itself at 5%? at 10%? at 20%?

10. The amount of a principal for a certain time at 6% is \$260, and for the same time at 9% is \$290; required the principal and the time.

WRITTEN EXERCISES.

1. In what time will \$650 give \$136.50 interest at 6%?

SOLUTION.—The interest of \$650, at 6%, for *one* year is \$39. If in *one* year the principal gives \$39 interest, to give \$136.50 interest it will require as many times 1 year as \$39 is contained times in \$136.50, which is 3½ yr., or 3 yr. 6 mo. Hence we have the following

OPERATION.

$$\begin{array}{r l} \$650 & \\ \underline{.06} & \\ 39.00 & \text{Int. 1 yr.} \\ \frac{136.50}{39.00} & = 3\tfrac{1}{2}\text{ yr.} \\ & = 3\text{ yr. 6 mo.} \end{array}$$

Rule.—*Divide the given interest by the interest of the principal at the given rate for* ONE *year.*

NOTE.—When the amount is given, subtract the principal from the amount to find the interest, and then proceed as before.

In what time will

2. $840, at 6%, give $184.80 interest? *Ans.* 3 yr. 8 mo.
3. $350, at 6%, amount to $462? *Ans.* 5 yr. 4 mo.
4. $1080, at 5%, give $124.20 int.? *Ans.* 2 yr. 3 mo. 18 da.
5. $37.50, at 7½%, give $13.50 int.? *Ans.* 4 yr. 9 mo. 18 da.
6. $18.20, at 5¾%, give $10.23 int.? *Ans.* 9 yr. 9 mo. 9 da.
7. $1800, at 4½%, am't to $3047.40? *Ans.* 15 yr. 4 mo. 24 da.

8. The amount of a certain principal in a certain time, at 6%, is $964.80, and the amount for the same time at 10% is $1128; required the principal and the time.

Ans. Prin., $720; Time, 5 yr. 8 mo.

SUG.—The difference of the amounts equals the interest at 4%.

9. A certain sum of money on interest amounts, at 8%, for a certain time, to $1540, and at 12%, for the same time, to $1810; required the principal and time.

Ans. Prin., $1000; Time, 6 yr. 9 mo.

472. Given the principal, the time, and the interest or the amount, to find the rate.

ORAL EXERCISES.

1. At what rate will $75 in 4 years give $27 interest?

SOLUTION.—For 4 years at *one* per cent., $\frac{4}{100}$ or $\frac{1}{25}$ of the principal equals the interest; $\frac{1}{25}$ of $75 equals $3; if $75 in 4 years at *one* per cent. gains $3, to gain $27 it will require as many times 1 per cent. as $3 is contained times in $27, which are 9 per cent.

At what per cent. will

2. $60 in 5 yr. give $18 int.?
3. $50 in 6 yr. give $15 int.?
4. $80 in 7 yr. give $28 int.?
5. $12 in 5 yr. give $15 am't?
6. $90 in 5 yr. give $117 am't?
7. $50 in 3 yr. 8 mo. give $61 am't?

495536

8. At what rate will a principal gain 2 times itself in 30 years? 4 times itself? 5 times itself?

9. At what rate will a principal double itself in 10 years? in 12 yr.? in 20 yr.? in 25 yr.?

10. At what rate will a principal treble itself in 20 years? in 25 yr.? in 40 yr.? in 80 yr.?

11. The amount of a certain principal for 5 years at a certain rate per cent. is $390, and for 8 yr. is $444; required the principal and the rate per cent.

WRITTEN EXERCISES.

1. At what rate will $650 give $136.50 interest in 3 yr. 6 mo.?

SOLUTION.—We find the interest of $650 for 3 yr. 6 mo. at *one* per cent. is $22.75. If the principal in the given time, at *one* per cent., gives $22.75 interest, to give $136.50 interest it will require as many times 1 per cent. as $22.75 is contained times in $136.50, which is 6%. Hence we have the following

OPERATION.
$650
.01
6.50
3½
$22.75 = Int. at 1%
$136.50 ÷ $22.75 = 6

Rule.—*Divide the given interest by the interest of the principal for the given time at* ONE *per cent.*

NOTE.—When the amount is given, subtract the principal from the amount to find the interest, and proceed as before.

At what rate will

2. $280 in 4 yr. 6 mo. give $63 interest? *Ans.* 5%.

3. $864 in 5 yr. 4 mo. give $368.64 interest? *Ans.* 8%.

4. $87.60 in 4 yr. 3 mo. 12 da. give $22.5132 int.? *Ans.* 6%.

5. $3975 in 6 yr. 7 mo. 20 da. give $2375.06¼ int.? *Ans.* 9%.

6. $15.60 in 9 yr. 10 mo. 15 da. give $12.324 int.? *Ans.* 8%.

7. The amount of a certain principal for 7 yr. at a certain rate is $4470, and for 11 yr. at the same rate it is $5310; required the principal and the rate. *Ans.* $3000; 7%.

8. The amount of a certain principal for 6 yr. at a certain rate per cent. is $3775, and for 18 yr. $6325; required the principal and rate. *Ans.* $2500; 8½%.

DISCOUNT AND PRESENT WORTH.

473. **Discount** is an allowance made for the payment of money before it becomes due.

474. The **Present Worth** of a debt payable at a future time without interest is such a sum as, being on interest for the time at a certain rate, will amount to the debt.

475. The **True Discount** is the difference between the amount of the debt and the present worth.

1. The *true discount* is the *interest* on the present worth for the time between the payment of the debt and the time it becomes due.

2. The present worth corresponds to the principal, the discount to the interest, and the debt to the amount; hence the different cases may be solved as in Interest.

1. What is the present worth of $620, payable 4 years hence, money being worth 6%?

SOLUTION.—The amount of $1 for 4 years at 6% is $1.24; hence the present worth of $1.24 is $1, and the present worth of $620 is as many times $1 as $1.24 is contained times in $620, which is $500. Hence the following

OPERATION.
$\$0.06 \times 4 = \$0.24$
Amount $= \$1.24$
$\$620 \div 1.24 = \$500$

Rule.—I. *To find the present worth, divide the given sum or debt by the amount of* $1 *for the given rate and time.*

II. *To find the discount, subtract the present worth from the given sum or debt.*

NOTE.—When several payments are made without interest, find the present worth of each separately, and take their sum.

WRITTEN EXERCISES.

2. What is the present worth of $484, payable 3 yr. 6 mo. hence, money worth 6%? *Ans.* $400.

3. What is the discount of $829, payable 2 yr. 7 mo. 18 da. hence, money worth 4%? *Ans.* $79.

4. What is the present worth of $1000, payable in 9 mo., money worth 6%? 7%? 8%? *Ans.* $956.94; $950.12; $943.40.

5. Mr. Johnson bought goods for $3000 on 3 mo. credit; what discount should he receive if he pays cash, money worth 6%? *Ans.* $44.34.

6. Mr. Watson, wishing to sell his house, offers it for $5720 on 3 years' credit, or $5000 cash; which would be most profitable for the buyer, money worth 4%? *Ans.* The latter, $107.14.

7. I gave a note for $1500, payable in 3 yr. 3 mo., but at the end of 9 mo. I wish to pay it; what should the holder receive, money worth 5%? *Ans.* $1333.33⅓.

BANK DISCOUNT.

476. A **Bank** is an incorporated institution, established for the purpose of receiving and loaning money or furnishing a paper circulation.

477. The money received by a bank for safe-keeping is called its *Deposits*, and the person placing money in a bank is called a *Depositor*.

478. A **Check** is an order on a bank, given by one of its depositors, to pay a certain amount to some person, or to his order, or to bearer.

479. Banks loan money on notes, mortgages, and other securities. Much of their business, however, consists in discounting notes, or paying them before they are due.

1. A person wishing to borrow money at a bank presents a note, either made or indorsed by himself, payable at a certain time, and receives for it a sum equal to its face, *less* the interest for the time the note has to run. This amount, called *discount*, is withheld by the bank in consideration of advancing money on the note prior to its maturity.

2. In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, the *day of discount* and *day of payment* are both reckoned. A 60-day note in Pennsylvania would be discounted for 61 days; in the others named (grace being allowed) for 64 days.

3. Business-men often discount notes by deducting the interest for a given time, with or without grace, as may be agreed upon. The rate is fixed by agreement, and may be other than the legal rate.

480. **Bank Discount** is the interest on the face of the note for the time from the day of discount to the day of payment.

481. The **Proceeds** or **Avails** of a note is the sum received for it when discounted, and equals the face (or amount) of the note less the discount.

482. The **Term of Discount** is the number of days from the time of discounting a note to the time of its maturity.

483. The difference between bank discount and true discount may be shown as follows:

If I take my note to the bank, promising to pay $106 at the end of 1 year, to get it cashed, by the method of true discount I would receive $100; but by the method of bank discount, not counting days of grace, I would receive $106 minus the interest of $106 for 1 year—that is, $106 - $6.36 = $99.64.

484. To find the discount, proceeds, or face from the other three elements.

1. What is the present worth or proceeds of a note for $450, due in 90 days, discounted at a bank at 6 per cent.?

SOLUTION.—The interest on $450 for 90 da., at 6%, is $6.75, which is the discount. Subtracting $6.75 from $450, we have $443.25, the proceeds.

OPERATION.

Int. on $1 = $.015

$450 × .015 = $6.75

$450 - $6.75 = $443.25

Rule.—I. *Find the interest on the face of the note for the time and rate, for the discount.*

II. *Subtract the discount from the face to find the present worth.*

1. The discount of an *interest-bearing* note is computed on the *amount* of the note at its maturity.

2. Banks compute interest for the *actual number of days* a note has to run, whether a note is drawn for months or days.

3. Days of grace are not reckoned in the following problems except in the 11th. In Exs. 10 and 11 both the day of discount and the day of payment are reckoned.

WRITTEN EXERCISES.

2. A note for $350, due in 90 days, was discounted at a bank at 6%; what was the discount? *Ans.* $5.25.

3. A note for $540, due in 60 days, was discounted at a bank, at 5%; required the proceeds. *Ans.* $535.50.

4. I wish to borrow $2000 from a bank for 90 days; for what must I give my note, discount 6%? *Ans.* $2030.46.

5. I presented a 60-da. note at a bank, which, discounted at 6%, netted me $5000; what was its face? *Ans.* $5050.50.

6. A broker buys a 60-da. note for $20 less than its face; what was the face, discount 6%? *Ans.* $2000.

7. A 3-mo. note for $600, dated June 1, was discounted July 10, at 5%; what was the discount? *Ans.* $4.42 —.

8. A 4-mo. note for $2000, dated Oct. 20, was discounted Dec. 16, at 8%; required the proceeds. *Ans.* $1970.67 —.

9. Find the difference between the true and the bank discount of $950 for 3 yr. 3 mo. at 6%. *Ans.* $30.23.

485. Find the results of the following notes:

10. $400. PHILADELPHIA, *Jan.* 5, 1895.

Ninety days after date, I promise to pay Charles Garrett, or order, Four Hundred Dollars, at the Girard Bank, for value received, without defalcation. JOHN WATERMAN.

Discounted, Jan. 10, at 6%.

Ans. April 5; 86 da.; dis., $5.73; proceeds, $394.27.

11. $\$354\frac{25}{100}$. WASHINGTON, D. C., *April* 3, 1895.

Six months after date, for value received, I promise to pay Edward Strong, or order, Three Hundred Fifty-four $\frac{25}{100}$ *Dollars, at the First National Bank.* JOHN MCPHERSON.

Discounted June 3, at 6%.

Ans. Oct. 3|6; 126 da.; dis., $7.44; proceeds, $346.81.

EXCHANGE.

486. **Exchange** is the method of making payments in distant places by means of *Drafts* or *Bills of Exchange.*

Suppose A of New York owes B of Chicago $800 for grain, and C of Chicago owes D of New York $800 for dry-goods. Then C of Chicago may go to B and pay him $800 for an order upon A, which he remits to D, and D collects the money from A. Both debts will thus be cancelled without sending any money, B receiving his $800 from C, and D his $800 from A.

The business of exchange is usually carried on by means of banks and bankers, who buy and sell drafts for any amount required. Exchange in the United States is mostly through the banks of the principal business centres.

487. A **Draft** or **Bill of Exchange** is a written order for the payment of money. In domestic exchange a bill is usually called a *Draft.*

1. The person who signs the order is called the *Maker* or *Drawer.*
2. The person requested to pay is called the *Drawee.*
3. The person to whom the money is to be paid is the *Payee.*
4. The person who has possession of the bill is the *Owner* or *Holder.*

488. The **Acceptance** of a draft is the promise of the Drawee, when presented, to pay it at maturity.

489. When the Drawee *accepts* the draft, he writes across the face, "Accepted," with the date and his signature; the bill is then called an *Acceptance,* and is of the character of a promissory note.

1. If a bill is protested for non-acceptance, the maker is under obligations to pay it immediately, although the time specified in it has not expired. When a bill reads "Acceptance waived," it is not subject to protest until maturity.

2. Bills of exchange are entitled to "days of grace" according to the custom of the place where the draft is payable, unless a particular day is named. In New York, Pennsylvania, etc. no grace is allowed, and in most States no grace is allowed on sight drafts.

490. There are two kinds of exchange—*Domestic Exchange* and *Foreign Exchange.*

DOMESTIC EXCHANGE.

491. **Domestic** or **Inland Exchange** is the exchange between two places in the same country.

492. The **Rate of Exchange** is the rate per cent. which is reckoned upon a draft.

The *Course of Exchange* is the current price paid in one place for bills of exchange upon another. The *brokerage* is usually included in the quotation of exchange.

493. Exchange is at *par* when a draft or bill sells for its face; at a *premium* when it sells for more than its face; and at a *discount* when it sells for less than its face.

1. The rate of exchange between two places depends upon the "course of trade." If the trade between New York and Chicago is equal, exchange is at par. If New York owes Chicago, the demand in New York for drafts on Chicago is greater than the demand in Chicago for drafts on New York, hence the drafts are at a *premium* in New York. But if Chicago owes New York, the demand for drafts is less in New York than in Chicago; hence drafts in New York on Chicago are at a *discount*.

2. The reason why the banks in New York should charge a premium is, that they must be at the expense of actually sending money to the Chicago banks or be charged with interest on their unpaid balance; the reason why New York banks will sell at a discount in the second case is that they are willing to sell for less than the face of a draft in order to get the money owed them in Chicago immediately.

494. A **Sight Draft** is one payable "at sight," or on its presentation. A *Time Draft* is one payable at a specified time after sight or date.

495. The **Forms** and **Use** of drafts may be seen by the following examples and explanations:

NATIONAL UNION BANK OF MARYLAND,
$5000. BALTIMORE, MD., *Jan.* 16, 1895.

At sight, pay to the order of Thomas Brown, Five Thousand Dollars, value received. WM. H. WELLS,
To the PHILADELPHIA NATIONAL BANK, *Cashier.*
PHILADELPHIA, PA.

EXPLANATION.—Suppose Thomas Brown of Baltimore owes John Farnum & Co. of Philadelphia $5000; he goes into a bank in Baltimore and gets the above draft. He then writes on the back of the draft, "Pay to the order of John Farnum & Co.," signing his name, and forwards it to John Farnum & Co. in Philadelphia, who take it to the Philadelphia National Bank, and, writing the name of their firm on the back, receive the money.

MERCHANTS' NATIONAL BANK,
$3400. CINCINNATI, O., *June* 20, 1891.

At ten days' sight, pay to the order of Hilton, Hughes & Co., Three Thousand Four Hundred Dollars, value received, and charge to account of C. A. STEVENS,
To the MERCHANTS' NATIONAL BANK, *Cashier.*
NEW YORK.

EXPLANATION.—Suppose that James Elliott of Cincinnati, wishing to pay a debt of $3400 to Hilton, Hughes & Co. of New York, buys the above draft on the Merchants' National Bank of New York. He forwards it to Hilton, Hughes & Co., who, having indorsed it, will present it at the bank. The "ten days' sight" means ten days after acceptance. It should be presented to the bank upon which it is drawn as soon as received, when the cashier writes upon it "Accepted," with the date of acceptance, and signs his name as cashier. This makes the bank liable for its payment, and is an agreement to pay it after ten days.

496. To find the cost or the face of a bill of exchange at sight or on time.

1. What must I pay in Chicago for a draft of $500 on Savannah, exchange being ½ per cent. discount?

SOLUTION.—Since exchange on Savannah is ½% discount, the cost of $1 is $1 − ½ ct. = $.99½, and the cost of $500 is 500 times $.99½, or $497.50. Hence for sight exchange we have the following

OPERATION.
$1.000
.005 = rate of exchange.
.995 = cost of $1.
500
$497.500, *Ans.*

Rule.—*Find the cost of* $1 *by adding the rate to* $1 *when at a premium, or subtracting it when at a discount, and multiply the result by the face of the draft.*

2. What must be paid in Detroit for a draft of $3000 on Boston at 30 days, exchange being ¼% premium?

Solution.—The draft, being on time, should be purchased at a discount. The discount of \$1, at the legal rate in Michigan, for 30 + 3, or 33 da., is \$.0055, which, subtracted from \$1, equals \$.9945, the cost of \$1 of the draft at par; but the premium is ¼%, hence, adding \$.0025, we find the actual cost of \$1 of the draft to be \$.9970, and multiplying this by \$3000, we have \$2991, the entire cost. Hence, for time exchange the following

OPERATION.

\$1.0000
.0055 = discount for 33 da.
.9945 = cost of \$1 at par.
.0025 = rate of exchange.
.9970 = cost of \$1 of draft.
3000
\$2991 = whole cost.

Rule.—*From \$1 subtract the bank discount of \$1 for the time and rate where the draft is purchased; to this result add the rate of exchange when at a premium, and subtract it when at a discount, and multiply the result by the face of the draft.*

WRITTEN EXERCISES.

3. Find the cost of a Boston draft on Chicago for \$2500 at 75¢ discount per \$1000. *Ans.* \$2498.125.

4. D. C. Heath & Co. of Boston owe a bill in Chicago of \$4375; what must they pay for a draft on Chicago, exchange ⅜% premium? *Ans.* \$4391.41 —.

5. Required the cost in New Orleans of a draft on Philadelphia for \$5000, payable 60 days after sight, at ¼% premium, rate 5%, with grace. *Ans.* \$4968.75.

6. A Baltimore firm received flour from Minneapolis costing \$3500; what must they pay for a draft at 15 days' sight, with grace, exchange ⅛% discount? *Ans.* \$3484.54 +.

7. A San Francisco merchant bought goods in New York valued at \$5284; what will be the cost of a 3 mo. draft for the amount on New York at ¾% premium? *Ans.* \$5231.16.

8. What is the face of a sight draft that can be bought for \$2587, exchange ½% discount?

Cost of \$1 = \$1.00 − \$.005 = \$0.995.
Face of draft = \$2587 ÷ \$.995 = \$2600.

9. I paid \$2029 for a 30-da. draft on Baltimore, interest 6%, premium 2%; what was the face of the draft?

Cost of \$1 = 1 − .0055 + .02 = \$1.0145.
Face of draft = 2029 ÷ 1.0145 = \$2000.

10. What is the face of a 90-day draft on Philadelphia, bought for \$3500, at 6%, exchange $1\frac{1}{2}$% premium? *Ans.* \$3500.

11. I received from Chicago a check for \$40.20, which cost $\frac{3}{4}$% to have cashed; what should have been the face of the check that I might have realized \$40.20? *Ans.* \$40.50.

FOREIGN EXCHANGE.

497. Foreign Exchange is the exchange that takes place between different countries.

1. Three bills of the same tenor and date are usually drawn, each containing a condition that it shall continue payable only while the others are unpaid. They are called a *Set of Exchange.*

2. To prevent loss or delay each bill of a set is sent by a different mail. When one bill of the set has been paid the others are void.

3. *Bills of Exchange* are usually made payable either 3 days after sight or 60 days after sight. The latter are quoted at a lower rate on account of the discount.

4. Most of the dealings in foreign exchange are with the following financial centres: London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Bremen, Berlin, and Vienna.

498. A **Letter of Credit** is a letter from a banking-house in one country to one or more of their correspondents in other countries, by which the person in whose favor the letter is written may draw money at different times and in different places, the total amount not exceeding a certain sum specified in the letter.

1. A bill of exchange is payable at a certain place, at a certain time, and for a certain amount, while a letter of credit is payable at different places, at different times, and for different amounts.

2. In giving quotations of foreign exchange no reference is made to the par value, but the premium or discount is included in the figures given.

499. The **Money of Account** of any country consists of the denominations of the money of that country in which accounts are kept.

1. What is the cost of a bill of exchange on London for £225 at $4.87¾ to the pound sterling?

OPERATION.

$$\begin{array}{r} \$4.87\frac{3}{4} \\ 225 \\ \hline \$1097.43\frac{3}{4} \end{array}$$

Solution.—If £1 costs $4.87¾, £225 cost 225 times $4.87¾, which is $1097.43¾.

Rule.—*Find the cost of a unit of the currency in which the bill is given, and multiply the face by it for the cost or divide the cost by it for the face.*

WRITTEN EXERCISES.

2. A banker sold a bill of exchange on Hamburg, face 7200 marks, at 94⅛ cents = 4 marks; what was the amount received? *Ans.* $1694.25.

3. What will be the face of a bill of exchange on Paris that can be bought for $4822.762, exchange 5.18⅜ francs to $1? *Ans.* 25000 fr.—.

4. What is the face of a bill of exchange on Bremen that was purchased for $19818.75, exchange being quoted at 94⅜¢ = 4 marks? *Ans.* 84000 marks.

5. What will be the cost in New York of the following draft, exchange at 60 days being $4.89? *Ans.* $1467.

Exchange for £300. New York, *April* 20, 1894.

Sixty days after sight of this First of Exchange (second and third unpaid) pay to the order of John Smith, Three Hundred Pounds Sterling, for value received, and charge the same to account of Brown Brothers & Co.

To Brown, Shipley & Co., London, England.

6. What will it cost to remit 9750 francs to Antwerp if 19.5¢ = 1 franc? *Ans.* $1901.25.

INTRODUCTION TO PROPORTION.

ORAL EXERCISES.

1. What is the relation of 8 to 4? *Ans.* 8 is *two times* 4.

2. What is the relation of 12 to 3? Of 16 to 4? Of 18 to 6? Of 20 to 5? Of 24 to 6? Of 30 to 5?

3. What is the relation of 3 to 6? Of 4 to 12? Of 6 to 24? Of 7 to 35? Of 8 to 57? Of 9 to 62?

4. The measure of the relation of two numbers is called their *ratio.*

5. What is the ratio of 12 to 4? *Ans.* The ratio of 12 to 4 is *three.*

6. What is the ratio of 18 to 9? Of 25 to 5? Of 48 to 8? Of 63 to 7? Of 64 to 4? Of 70 to 10? Of 80 to 8?

7. What is the ratio of 3 to 6? *Ans.* The ratio of 3 to 6 is *one-half.*

8. What is the ratio of 4 to 12? Of 3 to 18? Of 4 to 30? Of 9 to 108? Of 11 to 132? Of 12 to 144?

9. What is the ratio of $\frac{1}{2}$ to $\frac{1}{3}$? Of $\frac{1}{5}$ to $\frac{1}{6}$? Of $\frac{2}{3}$ to $\frac{3}{4}$? Of .5 to .25? Of .2 to .04? Of .03 to .12?

10. The ratio of two numbers may be expressed by writing the colon between them; thus, 8 : 4 denotes the ratio of 8 to 4.

11. Required the value of 12 : 6; of 28 : 7; of 42 : 6; of 24 : 12; of 12 : 24.

12. How does the ratio of 8 to 4 compare with the ratio of 12 to 6? *Ans.* They are equal.

13. What number has the same ratio to 12 that 18 has to 6?

14. What number has the same ratio to 20 that 40 has to 10?

15. The ratio of 9 to 36 is the same as the ratio of 15 to what number?

16. 25 is to 5 as 40 to what number? 24 is to 12 as 15 is to what number?

17. When we express the ratio of two numbers equal to the ratio of two other numbers, as, 24 is to 4, as 36 is to 6, we have a *proportion.*

18. The equality of two ratios may be expressed by writing the symbol = between them; thus, 8 : 4 = 12 : 6.

19. How many numbers do we have in a proportion? How many ratios? Are the ratios equal or unequal?

20. Write the proportion 16 to 8 as 24 is to 12; also 15 is to 45 as 18 is to 54.

SECTION IX.

RATIO AND PROPORTION.

500. **Ratio** is the measure of the relation of two similar quantities; thus, the ratio of 8 to 4 is 2.

501. The **Symbol** of ratio is the colon (:); thus, 8 : 4 signifies the ratio of 8 to 4. Ratio is also expressed by writing the numbers in the form of a fraction; thus, $\frac{8}{4}$.

502. The **Terms** of a ratio are the two numbers compared, called respectively the *antecedent* and the *consequent.*

503. The **Antecedent** is the number compared with the consequent. Thus, in the ratio 8 : 4, 8 is the antecedent.

504. The **Consequent** is the number with which the antecedent is compared. Thus, in 8 : 4, 4 is the consequent.

505. A **Ratio** is found by dividing the antecedent by the consequent. Thus, in 8 : 4 the ratio is $\frac{8}{4}$, or 2.

506. A **Simple Ratio** is the ratio of two numbers, as 6 : 3. A *Compound Ratio* is the product of two or more simple ratios; as, $(3:4)\times(5:6)$, or $\frac{3}{4}\times\frac{5}{6}$.

507. A **Compound Ratio** is usually expressed by writing the simple ratios one under another; thus, $\left\{\begin{matrix}3:4\\5:6\end{matrix}\right\}$.

508. Ratio exists only between similar quantities, and is always an abstract number.

1. The symbol of ratio (:) is supposed to be a modification of the symbol of division.

2. Ratio is usually defined as the *relation* of two numbers. This is indefinite, for the ratio is the *measure* of the relation.

PRINCIPLES.

1. *The ratio equals the quotient of the antecedent divided by the consequent.*

Thus, if the antecedent is represented by a, and the consequent by c, and the ratio by r, we have $a \div c = r$, or $\frac{a}{c} = r$.

2. *The antecedent is equal to the product of the consequent and ratio.*

For, since $\frac{a}{c}=r$, multiplying by c, we have $a=c\times r$.

3. *The consequent is equal to the quotient of the antecedent divided by the ratio.*

For, since $\frac{a}{c}=r$, $a=c\times r$, from which we see that $c=\frac{a}{r}$.

ORAL AND WRITTEN EXERCISES.

What is the ratio of

1. 8 to 2? *Ans.* 4.

2. 18 to 3? *Ans.* 6.

3. 75 to 12? *Ans.* $6\frac{1}{4}$.

4. 496 to 31? *Ans.* 16.

5. \$288 to \$648? *Ans.* $\frac{4}{9}$.

6. £144 : £256? *Ans.* $\frac{9}{16}$.

7. $\frac{1}{2}:\frac{5}{8}$? $\frac{2}{3}:\frac{5}{6}$? *Ans.* $\frac{4}{5}$.

8. $\frac{25}{16}:\frac{15}{12}$? $2\frac{3}{4}:2\frac{1}{5}$? *Ans.* $1\frac{1}{4}$.

9. What is the value of the compound ratio $\left\{\begin{matrix}3:6\\4:12\end{matrix}\right\}$.

SOLUTION.—This compound ratio equals $(3:6)\times(4:12)$, which equals $\frac{3}{6}\times\frac{4}{12}=\frac{1}{6}$.

10. What is the value of the ratio $\left\{\begin{matrix}5:3\\4:8\end{matrix}\right\}$? *Ans.* $\frac{5}{6}$.

11. What is the value of the ratio $\left\{\begin{matrix}3:2\\8:6\end{matrix}\right\}$? *Ans.* 2.

12. The antecedent is 15, the consequent 5; what is the ratio? *Ans.* 3.

13. The consequent is 7 and ratio 10; what is the antecedent? *Ans.* 70.

14. The antecedent is 56 and ratio 4; what is the consequent? *Ans.* 14.

15. The consequent is $\frac{19}{20}$ and ratio $\frac{25}{38}$; what is the antecedent? *Ans.* $\frac{5}{8}$.

16. The antecedent is $\frac{27}{40}$ and ratio $\frac{25}{32}$; what is the consequent? *Ans.* $\frac{108}{125}$.

17. Can you express the ratio between \$24 and 6 lb.? Why not?

18. The antecedents of a ratio are 7 and 8, and the consequents 12 and 18; what is the ratio? *Ans.* $\frac{7}{27}$.

SIMPLE PROPORTION.

509. A **Proportion** is the expression of equality between equal ratios, the terms of the ratios being indicated.

Thus, the ratios 8 : 4 and 12 : 6 being equal, they will form a proportion.

510. A Proportion is *written* by placing the sign = or the double colon (: :) between the two ratios.

Thus, $8 : 4 = 12 : 6$, or $8 : 4 :: 12 : 6$.

511. A Proportion is *read* in two ways; thus, $8 : 4 :: 6 : 3$ is read "the ratio of 8 to 4 equals the ratio of 6 to 3," or "8 is to 4 as 6 is to 3."

512. The **Terms** of a proportion are the four numbers used in the comparison. The first and fourth terms are the *Extremes;* the second and third are the *Means.*

513. The **Couplets** are the two ratios compared. The *first couplet* consists of the first and second terms; the *second couplet* consists of the third and fourth terms.

514. A **Simple Proportion** is the expression of the equality of two simple ratios.

515. The **Principles** of proportion are the truths relating to a proportion. They enable us to find any one term when the other three are given.

PRINCIPLES.

1. *In every proportion the product of the means equals the product of the extremes.*

In any proportion, as $6 : 3 = 8 : 4$, we have $\frac{6}{3} = \frac{8}{4}$, and multiplying these equals by 4 and 3, we have $6 \times 4 = 8 \times 3$; that is, the product of the two means, 8 and 3, equals the product of the two extremes, 6 and 4.

2. *Either extreme equals the product of the means divided by the other extreme.*

For, from the proportion $6 : 3 = 8 : 4$ we have $6 \times 4 = 3 \times 8$; hence, $6 = 3 \times 8 \div 4$, or $4 = 3 \times 8 \div 6$. Therefore, etc.

3. *Either mean equals the product of the extremes divided by the other mean.*

For, from the proportion $6 : 3 = 8 : 4$ we have $6 \times 4 = 3 \times 8$; hence, $3 = 6 \times 4 \div 8$, or $8 = 6 \times 4 \div 3$. Therefore, etc.

1. Let the pupils be required to demonstrate these principles by using symbols of any numbers; that is, by letters. French authors usually represent the unknown term by x; the same is done in this work.

2. Principle 1 may be demonstrated by showing that in a proportion we have 2d term × ratio : 2d term : : 4th term × ratio : 4th term; in which we see the factors in the means are the same as the factors in the extremes.

INTRODUCTORY EXERCISES.

1. Write a proportion and point out the different terms and couplets. Write a proportion and show that the ratios are equal.

2. If we multiply the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

3. If we divide the antecedent of one couplet, what must we do to the other couplet to make the ratios equal?

4. Write a proportion and illustrate Prin. 1; Prin. 2; Prin. 3.

5. Show that if we change the two means one for the other, or the two extremes, the four numbers will still form a proportion.

6. Take some proportion, and show that we may invert the terms of the couplets and the four terms will still be in proportion.

WRITTEN EXERCISES.

Find the terms denoted by x in each of the following proportions:

1. $x : 12 = 5 : 20$. *Ans.* 3.
2. $x : 9 = 13 : 39$. *Ans.* 3.
3. $5 : 15 = 9 : x$. *Ans.* 27.
4. $15 : 45 = 16 : x$. *Ans.* 48.
5. $32 : x = 21 : 63$. *Ans.* 96.
6. $x : \frac{5}{9} : : \frac{3}{5} : \frac{7}{8}$. *Ans.* $\frac{8}{21}$.
7. $9 : 7.2 : : 3.5 : x$. *Ans.* 2.8.
8. $4.5 : x : : 8\frac{1}{2} : 55\frac{1}{4}$. *Ans.* $29\frac{1}{4}$.
9. $72 : 27 : : x : 15$. *Ans.* 40.
10. \$8 : \$20 : : x : 75. *Ans.* 30.
11. \$21 : \$84 : : 65 lb. : x. *Ans.* 260 lb.

APPLICATION OF SIMPLE PROPORTION.

516. Simple Proportion is employed for the solution of problems in which three of four quantities are given, so related that the fourth may be determined from them by equality of the ratios.

517. Principle.—*The required quantity must bear the same relation to a given quantity of the same kind that one of the remaining quantities does to the other.*

518. From this principle we can form a proportion containing one unknown quantity, and find the unknown term by the principles of proportion.

NOTE.—Proportion was formerly called the "Rule of Three." Some of the old arithmeticians thought so highly of it that they called it "The Golden Rule of Three."

1. What will 25 tons of hay cost if 6 tons cost $36?

SOLUTION.—It is evident that the cost of 25 T. bears the same relation to the cost of 6 T. as 25 T. bears to 6 T.; hence, we have the proportion, *cost of* 25 T. *is to* $36 *as* 25 T. *is to* 6 T.; from which, by Prin. 2, we have the cost of 25 T. $= \frac{25 \times 36}{6} = \$150$. Hence the

OPERATION.

$ T. T.

Cost of 25 T. : 36 = 25 : 6

Cost of 25 T. $= \frac{25 \times 36}{6} = \$150$

Rule.—I. *Write the required quantity for the first term and the similar known quantity for the second term.*

II. *Place the other two quantities for the third and fourth terms, so that the two ratios will be equal.*

III. *Find the first term by dividing the product of the second and third terms by the fourth.*

SOLUTION 2D.—It is evident that the relation of 6 T. to 25 T. is the same as the relation of the cost of 6 T. to the cost of 25 T.; hence, we have the proportion 6 T. is to 25 T. as $36 is to the *cost of* 25 T., from which, by Prin. 2, we have the *cost of* 25 T. equals $150.

OPERATION.

T. T. $

6 : 25 = 36 : cost of 25 T.

Cost of 25 T. $= \frac{25 \times 36}{6} = 150$

1. The author believes that the simplest method of using proportion is to put the *unknown* quantity in the *first term*. He gives the old method also for teachers who prefer it. The above rule may be readily changed to correspond to it.

2. Pupils should be required to put the unknown quantity, which they may represent by x, in different terms, that they may thoroughly understand the subject.

WRITTEN EXERCISES.

2. If 15 yards of silk cost $22.50, what will 65 yards cost? *Ans.* $97.50.

3. If 56 yards of cloth cost $182, how many yards can be bought for $156? *Ans.* 48 yd.

4. If $100 gains $4 in a year, what will $450 gain in a year? *Ans.* $18.

5. If I can buy 80 horses for $10,000, how many can I buy for $37,500? *Ans.* 300.

6. What is the time by rail from Philadelphia to New York, 90 m., at the rate of 5 miles in 7 min. 30 sec.? *Ans.* 2¼ h.

7. A merchant gains $350 by selling $7000 worth of goods; what amount must he sell to gain $5000? *Ans.* $100,000.

8. If 5 A. 120 P. of land cost $718.75, what will 25 A. 40 P. cost? *Ans.* $3156.25.

9. A bankrupt's debts are $4800 and assets $2000; what will be received for a claim of $1500? *Ans.* $625.

10. If 35 oxen eat 45 acres of grass in a month, how many oxen would 540 acres keep the same time? *Ans.* 420 oxen.

11. If the tax on property valued at $4800 is $88.80, what will be the tax on property valued at $7600? *Ans.* $140.60.

12. If a pole 4 ft. 6 in. long throws a shadow of 5 ft. 3 in., what will be the shadow of a tree 54 ft. high? *Ans.* 63 ft.

13. If a pole 10 ft. long casts a shadow of 8 ft. 4 in., what is the height of a steeple whose shadow is 240 feet long? *Ans.* 288 ft.

14. A grocer has a false balance which gives 15 oz. to the pound; what does he gain by it in selling sugar for which he receives $74.40? *Ans.* $4.65.

15. If a man spends $276 in the three summer months, how much will he spend in a year at the same rate per day? *Ans.* $1095.

16. If 12 men build a wall in 24 days, how long will it take 60 men to build it at the same rate? *Ans.* $4\frac{4}{5}$ days.

NOTE.—Here it is evident that the time in which 60 men do it is to 24 days, the time in which 12 men do it, as 12 men is to 60 men.

17. If 28 men mow a field of grass in 12 days, how many men will be required to mow it in 8 days? *Ans.* 42 men.

18. If 17 men can mow a field in 9 days, how long would it take to mow half of it if 5 men refuse to labor? *Ans.* $6\frac{3}{8}$ da.

19. If it requires 54 yards of carpet 1 yd. wide to cover a floor, how many yards of carpet $\frac{3}{4}$ yd. wide will cover the same floor? *Ans.* 72 yd.

20. If a man performs a journey in 25 days of 8 hours each, how many days, of 10 hours each, would it take him to perform the same journey? *Ans.* 20 days.

21. A merchant borrows $1800 from a friend, and keeps it 1 yr. 6 mo.; how long should he loan $1000 to return the favor? *Ans.* 2 yr. 8 mo. 12 da.

22. Two cog-wheels, one having 30 and the other 25 cogs, run together; in how many revolutions of the larger wheel will the smaller gain 15 revolutions? *Ans.* 75.

23. If a 3-cent loaf weighs 9 ounces when flour is $6 a barrel, what should it weigh when flour is $4 a barrel? *Ans.* $13\frac{1}{2}$ oz.

24. A house is worth $2700, and the price of the house is to that of the lot on which it stands as 9 to 11; what is the value of the lot? *Ans.* $3300.

25. If 500 laborers can build an embankment in 36 days, how many more days would be required if the number of men is diminished by 50? *Ans.* 4 days more.

26. A can do a piece of work in 8 days; B can do it in 10 days; what should A receive per day if B's wages are $2 per day? *Ans.* $$2\frac{1}{2}$.

27. A has grain which cost 55¢ a bushel, and B has flour at $4.25 a barrel; if in exchange A puts his grain at 65¢ a bushel, what should B charge for his flour? *Ans.* $$5.02\frac{3}{11}$.

28. A garrison of 4000 men has "hard tack" enough for 8 weeks, allowing each man 16 oz. a day; but 14,000 lb. having been spoiled, what must be each man's allowance that the provision may last the 8 weeks? *Ans.* 15 oz.

29. How large a reinforcement could the above garrison receive if none of the provision was spoiled, but each man's allowance was reduced to $14\frac{2}{9}$ oz.? *Ans.* 500.

COMPOUND PROPORTION.

519. A **Compound Proportion** is a proportion in which one or both ratios are compound.

520. Thus, $\left\{\begin{matrix}2:4\\5:15\end{matrix}\right\} = 6:36$ and $\left\{\begin{matrix}4:12\\7:14\end{matrix}\right\} = \left\{\begin{matrix}5:10\\6:18\end{matrix}\right\}$, are examples of compound proportion.

521. Compound Proportion is used in the solution of problems in which the required term depends on a compound ratio.

522. In simple proportion the unknown quantity depends upon *one pair* of similar quantities; in compound proportion it depends upon *two or more pairs* of similar quantities.

523. The method of **Compound Proportion** is now seldom used in arithmetical operations.

524. The simplest method of solving the problems that belong to **Compound Proportion** is that of *Analysis.*

WRITTEN EXERCISES.

1. If 6 men can earn $90 in 5 days, how much can 8 men earn in 9 days?

ANALYSIS.—If 6 men earn $90 in 5 da., 1 man will earn $\frac{1}{6}$ of $90 and 8 men will earn $\frac{8}{6}$ of $90. If 8 men earn $\frac{8}{6}$ of $90 in 5 da., in 1 day they will earn $\frac{1}{5}$ of $\frac{8}{6}$ of $90, and in 9 da. they will earn $\frac{9}{5}$ of $\frac{8}{6}$ or $90, which, by cancelling and multiplying, equals $216.

OPERATION.

$$\overset{3}{\frac{\cancel{9}}{\cancel{5}}} \times \overset{4}{\frac{\cancel{8}}{\underset{\cancel{2}}{\cancel{6}}}} \times \overset{18}{\$\cancel{9}\cancel{0}} = \$216$$

525. The analysis may be abbreviated as follows:

Analysis.—If 6 men earn \$90, 8 men will earn $\frac{8}{6}$ of \$90. If they earn it in 5 da., in 9 da. they will earn $\frac{9}{5}$ of $\frac{8}{6}$ of \$90 = \$216.

2. If it costs \$42 to carpet a floor 24 feet long and 15 feet wide, what will it cost to carpet a floor 30 feet long and 18 feet wide? *Ans.* \$63.

3. If a railroad charges \$15 for carrying 12000 lb. 300 miles, what should be the charge for carrying 50000 lb. 250 miles? *Ans.* \$52$\frac{1}{12}$.

4. If it costs \$1.45 for burning 5 gas-burners 5 hours every evening for 6 days, how many burners may be used 4 hours every evening for 10 days, at a cost of \$6.96? *Ans.* 18.

5. If 10 men in 48 days build a wall 140 rd. long, 6 ft. high, and 4 ft. thick, how many men can in 54 days build a wall 180 rd. long, 7 ft. high, and 4$\frac{1}{2}$ ft. thick? *Ans.* 15 men.

6. If 15 carpenters build a house in 60 days, working 10 hours a day, in how many days would 30 carpenters build it, working 8 hours a day? *Ans.* 37$\frac{1}{2}$ days.

7. If 25 horses can eat a certain quantity of grain in 40 days, in what time will twice as much grain be consumed if 7 horses are added when the grain is $\frac{2}{3}$ eaten? *Ans.* 74$\frac{1}{6}$ days.

8. If 240 loaves of bread, weighing 6 oz. each, cost \$12 when flour is \$6 a barrel, what cost 100 loaves of 7 oz. each when flour is worth \$4.50 a barrel? *Ans.* \$4.37$\frac{1}{2}$.

9. If 22 men can dig a ditch 2500 ft. long, 5 ft. wide, and 3 ft. deep in 25 days of 10 hours each, in how many days of 11 hours each will 132 men dig a ditch 3600 ft. long, 6 ft. wide, and 4 ft. deep? *Ans.* 8$\frac{8}{11}$ days.

10. If 7 compositors in 17 days of 10 hours each set up 35 sheets of 16 pages each, 51 lines on a page and 45 letters in a line, in how many days, 12 hours long, can 6 compositors set up in the same type 27 sheets, 24 pages each, 45 lines in a page, 48 letters in a line? *Ans.* 18 days.

PARTNERSHIP.

526. **Partnership** is the association of two or more persons for the transaction of business.

527. The association is termed a *Firm*, *House*, or *Company*, and the persons associated are termed *Partners.*

528. The **Capital** of a firm is the money or property invested by the partners.

529. The **Resources** or **Assets** of a firm are its property of any kind, together with the amounts due it.

530. The **Liabilities** are its debts. The excess of resources over liabilities is called the *Net Capital.*

Partners are of three kinds, *General*, *Limited*, and *Special.*

General Partners risk their whole property in the business; *Limited and Special Partners* risk only the amount of capital they agree to contribute. Partners whose names do not appear are sometimes called *Silent Partners.*

531. **Partnership** is divided into *Simple* and *Compound Partnership* for convenience of treatment.

SIMPLE PARTNERSHIP.

532. In **Simple Partnership** the shares of the partners are employed for equal periods of time.

1. A, B, and C formed a partnership; A put in \$1000, B put in \$1200, and C put in \$1400; they gained \$900; what was each one's share of the gain?

SOLUTION.—The entire capital is \$3600. Since A put in \$1000, he furnished $\frac{1000}{3600}$, or $\frac{5}{18}$ of the capital, and hence should have $\frac{5}{18}$ of \$900, or \$250. B furnished $\frac{1200}{3600}$, or $\frac{1}{3}$ of the capital, and should have $\frac{1}{3}$ of \$900, or \$300, etc.

OPERATION.

$$\begin{array}{rl}
\$1000 & \frac{1000}{3600}=\frac{5}{18}=\text{A's share.}\\
1200 & \frac{1200}{3600}=\frac{1}{3}=\text{B's share.}\\
\underline{1400} & \frac{1400}{3600}=\frac{7}{18}=\text{C's share.}\\
\text{Stock}=\$3600 &
\end{array}$$

$$\begin{array}{r}
\frac{5}{18}\text{ of }\$900=\$250=\text{A's share.}\\
\frac{1}{3}\text{ of }\$900=\$300=\text{B's share.}\\
\frac{7}{18}\text{ of }\$900=\$350=\text{C's share.}
\end{array}$$

Rule.—*Divide the gain or loss among the partners in proportion to their shares of the stock.*

WRITTEN EXERCISES.

2. A, B, and C form a partnership for raising oranges: A puts in $560, B $640, and C $800; they gain $500; what does each receive? *Ans.* A, $140; B, $160; C, $200.

3. Three men enter into partnership with a capital of $15,000; A contributes ½ of the capital, B ⅓ of the capital, and C the remainder; what does each receive if they gain $7500? *Ans.* A, $3750; B, $2500; C, $1250.

4. A, B, and C go into the grain business with a capital of $12,000; at the end of a year A's share of the profits is $1250, B's $1000, and C's $750; what was each one's capital? *Ans.* A's $5000; B's $4000; C's $3000.

5. A man leaves by his will $5000 to his wife, $4000 to his son, and $3500 to his daughter; after his death his estate was found to amount to only $10,000; what did each receive? *Ans.* Wife, $4000; son, $3200; daughter, $2800.

6. A shipping-firm gained in one year $4000; A's stock was $7000, B's $4500, and C's gain was $406.25; what were A's and B's gain and C's stock?

Ans. C's stock, $1300; A's gain, $2187.50; B's, $1406.25.

COMPOUND PARTNERSHIP.

533. In **Compound Partnership** the capitals of the partners are employed for different periods of time.

1. Two persons enter into partnership and gain $560; A put in $400 for 12 mo., and B $800 for 8 mo.; what was each man's share of the gain?

SOLUTION.—$400 for 12 months is equivalent to $4800 for 1 mo., and $800 for 8 mo. is equivalent to $6400 for 1 mo.; hence, the entire capital is equivalent to $11200 for 1 mo. The rest of the solution may be given as in Simple Partnership.

OPERATION.

$\$400 \times 12 = \$4800$ for 1 mo. = A's.
$\$800 \times \ 8 = \underline{\$6400}$ for 1 mo. = B's.
$11200 for 1 mo.
$\frac{4800}{11200} = \frac{3}{7}$ = A's share of capital.
$\frac{6400}{11200} = \frac{4}{7}$ = B's share of capital.
$\frac{3}{7}$ of $\$560 = \$240$, A's gain.
$\frac{4}{7}$ of $\$560 = \$320$, B's gain.

Rule.—*Multiply each partner's capital by the time it was employed, and divide the gain or loss in proportion to these products.*

WRITTEN EXERCISES.

2. Three persons formed a partnership; A had $800 in trade for 9 mo., B $600 for 10 mo., and C $500 for a year; they gained $960; what was each man's share of the gain? *Ans.* A's, $360; B's, $300; C's, $300.

3. Three wood-choppers agree to cut the wood on a certain tract for $150: the first worked 15 days of 10 hours each, the second 25 days of 6 hours each, and the third 30 days of 5 hours each; what does each receive? *Ans.* $50.

4. Four men rented a pasture for $75.60; the first put in 4 horses for 6 weeks, the second 3 horses for 10 weeks, the third 5 horses for 8 weeks, and the fourth 2 horses for 16 weeks; what should each pay? *Ans.* $14.40; $18; $24; $19.20.

5. Three drovers hire a pasture for $144 for 6 mo.; the first put in 40 cows, the second 60, and the third 80; at the end of 4 mo. the first sold $\frac{1}{2}$ of his, the second $\frac{1}{3}$ of his, and the third $\frac{1}{4}$ of his; what should each pay? *Ans.* $30; $48; $66.

6. Mr. Warner went into business with $5000 capital; at the end of 4 months he took in Mr. Granville with $8000 capital, and at the end of 6 months Mr. Randolph with $3000 capital; at the end of the year they had gained $1420; what was the share of each? *Ans.* $600; $640; $180.

7. Two persons, A and B, were in partnership 2 years; A at first put in $4000 and B $3000; at the end of 9 mo. A took out $1200 and B put in $600; they lost in 2 yr. $3816; what was each one's share of the loss? *Ans.* A's, $1872; B's, $1944.

8. A and B went into the hardware business, A's capital being to B's as 3 to 4; at the end of 6 months A withdraws $\frac{1}{3}$ of his capital and B $\frac{1}{4}$ of his; during the year they gain $1440; what was each man's share of the gain? *Ans.* A's, $600; B's, $840.

EQUATION OF PAYMENTS.

534. Equation of Payments is the process of finding the mean or equitable time for paying several sums due at different times.

535. The **Term of Credit** is the time allowed for the payment of a debt.

536. The **Average Term of Credit** is the time to elapse before several debts due at different times may in equity be paid together.

537. The **Equated Time** is the date at which several debts due at different times may be paid in one sum.

538. The **Focal Date** is the date from which we begin the reckoning in averaging an account.

CASE I.

539. To find the average term of credit when the terms of credit begin at the same time.

1. A merchant owes at a bank $250, due in 5 months, and $450, due in 3 months; what is the average term of credit?

SOLUTION.—A credit on $250 for 5 months is regarded as equivalent to a credit on $1 for 1250 months, and a credit on $450 for 3 months is equivalent to a credit on $1 for 1350 months, and adding, we have the same as a credit on $1 for 2600 months; if $1 has a credit of 2600 months, $700 would have a credit of $\frac{1}{700}$ of 2600 months, which is $3\frac{5}{7}$ months. Hence the

OPERATION.

$250 \times 5 = 1250$

$450 \times 3 = 1350$

$\overline{700} \quad)\overline{2600}(3\frac{5}{7}$

Rule.—*Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts; the quotient will be the average term of credit.*

1. Cents in any of the payments may be rejected when less than 50, and reckoned at $1 when more than 50. The fraction of a day in the answer is also rejected when less than $\frac{1}{2}$, and reckoned as 1 day if more than $\frac{1}{2}$.

2. The time may also be found by dividing the sum of the interest on the payments, using any rate, by the interest on the sum of the payments for 1 month or 1 day, according to the unit of time used in the *culation.* This method is preferred by some accountants.

WRITTEN EXERCISES.

2. Henry Bowman owes the H. B. Claflin Co. \$4800, $\frac{1}{3}$ due in 3 mo., $\frac{1}{4}$ in 4 mo., and the remainder in 6 mo.; required the average term of credit. *Ans.* $4\frac{1}{2}$ mo.

3. T. B. Morgan owes \$1800, $\frac{1}{3}$ of which is due in 3 mo., \$600 in 6 mo., and the remainder in $8\frac{1}{4}$ mo.; required the average term of credit. *Ans.* $5\frac{3}{4}$ mo.

4. A. M. Stewart owes \$500 due in 4 mo., \$600 due in 6 mo., \$700 due in 8 mo., and \$1000 due in 9 mo.; what is the average term of credit? *Ans.* $7\frac{3}{14}$ mo.

5. I bought merchandise April 1, 1894, as follows: \$4500 for cash, \$1800 on 4 mo., and \$1200 on 6 mo.; what is the equated time of payment? *Ans.* May 29.

6. Mr. Woodruff bought a house, agreeing to pay $\frac{1}{2}$ in 6 mo., $\frac{1}{4}$ in 8 mo., and the remainder in 1 year; required the average term of credit. *Ans.* 8 mo.

REMARK.—Since the result will be the same whatever the sum owed, we may assume \$1 as the capital, and proceed as before.

CASE II.

540. To find the equated time when the terms of credit begin at different dates.

1. I purchased of A. S. Gardner & Co. the following bill of goods:

Mar. 10, a bill amounting to \$800 on 2 mo. credit.
Mar. 15, " " \$600 on 3 mo. "
Apr. 20, " " \$700 on 3 mo. "

Now, if I wish to make one payment of this bill, at what time in equity will it become due?

SOLUTION.—From the time the first is due to the time the second is due is 36 da, and to the time the third is due is 71 da.; hence, estimating from the time the first is due, the second has a credit of 36 da., and the third a credit of 71 da., and the first has a credit of no days.

OPERATION.

May 10, \$800 × 00 = 00000
June 15, \$600 × 36 = 21600
July 20, \$700 × 71 = 49700
2100)71300(34 –

We then average it as in Case I., and find the term of credit to be nearly 34 da. from May 10, the time at which the first debt is due; hence the equated time of payment is June 13. From the above we derive the following

Rule.—I. *Select the date at which the first debt becomes due, and multiply each debt by its term of credit reckoned from the date selected.*

II. *Divide the sum of the products by the sum of the debts, and the quotient will be the average term of credit, estimated from the date selected.*

When the *earliest* date is not the *first* of the month, it is often more convenient to take the *first* of the month as the standard date.

WRITTEN EXERCISES.

2. Bought goods on 60 days' credit as follows: Oct. 15, 1894, $200; Nov. 20, $360; Dec. 12, $155; required the equated time of payment of this bill. *Ans.* Jan. 14.

3. Bought goods on 4 mo. credit as follows: Jan. 12, 1893, $300; Feb. 15, $500; Apr. 4, $800; required the equated time of payment. *Ans.* July 4.

4. Sold goods on 6 mo. credit as follows: Feb. 16, 1893, $300; Mar. 12, $250; April 5, $350; June 1, $600; what is the equated time for the payment of this bill? *Ans.* Oct. 14.

5. Bought goods of Strawbridge & Clothier on 90 days' credit as follows: Mar. 4, $176.82; Apr. 7, $438.36; May 1, $632.65; June 1, $237.16; required the equated time for the payment of the bill. *Ans.* July 21.

6. F. A. Leggett & Co. bought of Lord & Taylor several bills of goods, as follows:

June 3, a bill of $375 on 30 days' credit.
" 28, " " $450 " 60 " "
July 16, " " $840 " 4 months' credit.
Sept. 12, " " $250 " 90 days' credit.

What is the equated time of payment? *Ans.* Oct. 5.

CASE III.

541. When a debt due at some future time has received partial payments, to find when the remainder should be paid.

1. A merchant bought goods to the amount of \$5000 on a credit of 6 mo.; 3 mo. before it was due he paid \$800, and 2 mo. before it was due he paid \$1200; how long after the expiration of the 6 mo. may the balance remain unpaid?

SOLUTION.—A credit on \$800 for 3 mo. is equivalent to a credit on \$1 for 2400 mo.; a credit on \$1200 for 2 mo. is equivalent to a credit on \$1 for 2400 mo.; and adding, we have a credit on \$1 for 4800 mo.; hence, \$3000, the sum which remains unpaid, should have a credit of $\frac{1}{3000}$ of 4800 mo., which is $1\frac{3}{5}$ mo. Hence

OPERATION.

$$\begin{array}{ll} 800 \times 3 = & 2400 \\ 1200 \times 2 = & \underline{2400} \\ \overline{2000} & 4800 \end{array}$$

$$\frac{4800}{3000} = 1\tfrac{3}{5}.$$

Rule.—*Multiply each payment by the time it was paid before it was due, and divide the sum of the products by the sum remaining unpaid.*

WRITTEN EXERCISES.

2. Mr. Jones borrowed \$3500 for 6 mo.: 4 mo. before it was due he paid \$800, and 3 mo. before due he paid \$1500; at what time in equity should the remainder be paid? *Ans.* $6\frac{5}{12}$ mo.

3. I lent Mr. Wilson \$2000 for 9 mo., ½ of which he paid in 4 mo., and ½ of the remainder in 6 mo.; how long in equity may the remainder remain unpaid? *Ans.* 13 mo. after due.

4. I borrowed of Mr. Carpenter \$500 for 3 mo., \$700 for 5 mo., and \$1000 for 6 mo.; at the end of 4 mo. I paid him \$1600; at what time in equity should the remainder be paid? *Ans.* $7\frac{2}{3}$ mo. after borrowing.

5. A milliner bought of Sharpless & Co. a bill of \$350 on a credit of 20 days, and \$480 on a credit of 30 days; at the end of 16 days she paid \$400, and at the end of 24 days she paid \$250; when, in equity, should the balance of the bill be paid? *Ans.* 50 days.

SECTION X.

INVOLUTION AND EVOLUTION.

INVOLUTION.

542. **Involution** is the process of finding any power of a number.

543. A **Power** of a number is the product arising from using the number several times as a factor. The number itself is called the *first power*.

544. The **Second Power** of a number is the product obtained by using the number twice as a factor. Thus, 16 is the second power of 4, since $4 \times 4 = 16$.

545. The **Third Power** of a number is the product obtained by using the number three times as a factor. Thus, 64 is the third power of 4, since $4 \times 4 \times 4 = 64$.

546. The **Fourth Power** of a number is the product obtained by using the number four times as a factor; the *Fifth Power*, five times as a factor, etc.

547. The **Degree** of a power is denoted by a small figure, called an *exponent*, placed at the right and a little above the number. Thus, 5^2 represents the 2d power of 5; 6^3, the third power of 6, etc.

548. The **Exponent** indicates how many times the number is used as a factor. Thus, 8^3 denotes that 8 is used as a factor three times; that is, $8 \times 8 \times 8$, which equals 512.

The second power of a number is called its *square*, because the area of a square equals the product of its two equal sides. The third power of a number is called its *cube*, because the product of the three equal sides of a cube gives its contents.

PRINCIPLES.

1. *A power of a number is obtained by using the number as* *actor as many times as there are units in the degree.*

2. *The product of any two powers of a number equals a power of the number denoted by the sum of the exponents.*

Thus, if we multiply the cube of a number by the 4th power of the number, we shall evidently have the number used seven times as a factor, or the 7th power of the number; thus, $5^3 \times 5^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^7$, and the same may be shown in any other case.

3. *A power of a number raised to any power equals a power of the number denoted by the product of the exponents.*

Thus, if we *square* the *cube* of a number, we shall evidently use the number as a factor *two* times *three* times, or *six* times; thus, $(5^3)^2 = 5^3 \times 5^3$, which, by Prin. 2, equals 5^6, and the same may be shown in any other case.

NOTE.—By means of this principle we can abbreviate the operation of involution; thus, we can raise a number to the 6th power by squaring its cube, or to the 12th power by squaring its 6th power, or cubing its 4th power, etc.

ORAL EXERCISES.

1. The square of 4 equals 4 used how often as a factor?

2. The cube of 5 equals 5 used how often as a factor?

3. How often is 8 used as a factor in finding the 5th power of 8?

4. How often is 5 used as a factor in the cube of the square of 5?

5. What power of 6 is 6^2 multiplied by 6^3?

6. If we multiply 7^3 by 7^4, what power of 7 shall we have?

7. What power of 5 is equal to 5^3 multiplied by 5^5?

8. What power of 2 is the square of the square of 2^3?

9. If the square of a number equals 6 times that number, what is the number?

10. What number multiplied by 8 gives ½ of the square of the number for a product?

11. What number multiplied by 16 gives ⅘ of the square of the number for a product?

WRITTEN EXERCISES.

1. Find the square of 15.

SOLUTION.—To find the square of 15 we multiply 15 by itself, and we have 225. To find the cube of 15 we would multiply 225 by 15.

OPERATION.

$$\begin{array}{r} 15 \\ \underline{15} \\ 225 \end{array}$$

2. Square 16. *Ans.* 256.
3. Square 24. *Ans.* 576.
4. Square 56. *Ans.* 3136.
5. Square 105. *Ans.* 11025.
6. Cube 12. *Ans.* 1728.
7. Cube 36. *Ans.* 46656.
8. Cube 63. *Ans.* 250047.
9. Cube 99. *Ans.* 970299.

Find the value of

10. 65^2. *Ans.* 4225.
11. 32^3. *Ans.* 32768.
12. 27^4. *Ans.* 531441.
13. $(\frac{16}{17})^2$. *Ans.* $\frac{256}{289}$.
14. $9^3 \times 9^4$. *Ans.* 9^7.
15. $8^4 \times 8^5$. *Ans.* 8^9.
16. $15^3 \times 15^5$. *Ans.* 15^8.
17. $(\frac{1}{2})^2 \times (\frac{1}{2})^3$. *Ans.* $\frac{1}{32}$.
18. $(6\frac{1}{4})^3$. *Ans.* $244\frac{9}{64}$.
19. $(3.8)^4$. *Ans.* 208.5136.
20. $(6.25)^3$. *Ans.* 244.140625.
21. $(10.5)^4$. *Ans.* 12155.0625.
22. $3^2 \times 3^3 \times 3^4$. *Ans.* 3^9.
23. $(\frac{7}{8})^3 \times (\frac{7}{8})^4$. *Ans.* $(\frac{7}{8})^7$.
24. $(2.5)^4 \times (2.5)^6$. *Ans.* $(2.5)^{10}$.
25. $(3.3)^2 \times (3.3)^3$. *Ans.* $(3.3)^5$.

SQUARING NUMBERS.

549. There are **Two Methods** of explaining the principles of squaring numbers, called the *Analytic or Algebraic*, and the *Synthetic* or *Geometrical.*

1. The object of these methods is to find the law of forming the square, and thus prepare for corresponding methods of explaining Evolution.

2. Teachers who prefer the geometrical method will find it in the Supplement.

NOTE.—Teachers who prefer the geometrical method of explaining evolution may omit involution by the analytic method.

1. Find the square of 45 analytically.

OPERATION.

$$\begin{array}{rl} 45 = & 40+5 \\ \underline{45} = & \underline{40+5} \\ 225 = & 5\times40+5^2 \\ \underline{180} = & \underline{40^2+\quad 5\times40} \\ 2025 = & 40^2+2\times(5\times40)+5^2 \end{array}$$

ANALYTICAL SOL.—Forty-five equals 40 plus 5, or 4 tens plus 5 units. Writing this as 40+5, and commencing at units to square, we have 5 times 5 equals 5^2, 5 times 40 equals 5×20, 40 times 5 equal 5×40, 40 times 40 equals 40^2, and adding, we have $40^2+2\times(5\times40)+5^2$; hence, the square of 45 equals the *square of the tens, plus twice the tens into the units, plus the square of the units,* which we find to be 2025.

WRITTEN EXERCISES.

Square the following numbers:

2. 24.	*Ans.* 576.	**8.** 254.	*Ans.* 64516.
3. 35.	*Ans.* 1225.	**9.** 467.	*Ans.* 218089.
4. 36.	*Ans.* 1296.	**10.** 703.	*Ans.* 494209.
5. 44.	*Ans.* 1936.	**11.** 838.	*Ans.* 702244.
6. 57.	*Ans.* 3249.	**12.** 2005.	*Ans.* 4020025.
7. 78.	*Ans.* 6084.	**13.** 4628.	*Ans.* 21418384.

550. The following principles, derived from the above solution, are important and should be committed to memory:

PRINCIPLES.

1. *The square of a number of two figures equals the* TENS^2 + 2 *times* TENS × UNITS + UNITS^2.

2. *The square of a number of three figures equals* HUNDREDS^2 + 2 *times* HUNDREDS × TENS + TENS^2 + 2 (HUNDREDS + TENS) × UNITS + UNITS^2.

551. These principles may also be expressed by letters. Let u represent units figure, t tens, h hundreds, and T thousands, and a period between two letters denote multiplication; then we have

$$(t+u)^2 = t^2 + 2t.u + u^2.$$

$$(h+t+u)^2 = h^2 + 2h.t + t^2 + 2(h+t).u + u^2.$$

$$(T+h+t+u)^2 = T^2 + 2T.h + h^2 + 2(T+h).t + t^2.$$

$$2(T+h+t).u + u^2.$$

CUBING NUMBERS.

552. There are **Two Methods** of explaining the principles of cubing numbers, called the *Analytic* or *Algebraic*, and the *Synthetic* or *Geometrical*, method.

1. The object of these methods is to find the law of forming the cube, and thus to prepare for corresponding methods of explaining Evolution.

2. Teachers who prefer the geometrical method will find it in the Supplement.

1. Find the cube of 45 by the analytical method.

ANALYTICAL SOL.—Squaring 45 by the method already given, we have $40^2+2\times(5\times40)+5^2$. We then multiply this by $40+5$. Five times 5^2 equals 5^3, 5 times $2\times5\times40$ equals $2\times5\times5\times40$, or $2\times5^2\times40$. Five times 40^2 equals 5×40^2. We next multiply by 40. Forty times $5^2=40\times5^2$, forty times $2\times5\times40$ equals $2\times5\times40^2$, forty times 40^2 equals 40^3. Taking the sum of these products, and we have first 5^3; next, *once* $5^2\times40$ plus *twice* $5^2\times40$ equals *three* times $5^2\times40$; next, *twice* 5×40^2 plus *once* 5×40^2 equals *three* times 5×40^2, and next we have 40^3; hence, $45^3=40^3+3\times5\times40^2+3\times5^2\times40+5^3$. Therefore the cube of 45 equals *the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units.*

OPERATION.

$$\begin{array}{lr}
45^2 = & 40^2+2\times(5\times40)+5^2\\
45 = & 40+5\\ \hline
 & 5\times40^2+2\times5^2\times40+5^3\\
 & 40^3+2\times5\times40^2+\quad 5^2\times40\\ \hline
45^3 = & 40^3+3\times5\times40^2+3\times5^2\times40+5^3
\end{array}$$

Cube the following numbers:

2. 14.	*Ans.* 2744.	**7.** 32.	*Ans.* 32768.
3. 17.	*Ans.* 4913.	**8.** 53.	*Ans.* 148877.
4. 21.	*Ans.* 9261.	**9.** 87.	*Ans.* 658503.
5. 25.	*Ans.* 15625.	**10.** 235.	*Ans.* 12977875.
6. 29.	*Ans.* 24389.	**11.** 327.	*Ans.* 34965783.

PRINCIPLES.

1. *The cube of a number consisting of two figures equals* TENS^3 + 3 *times* TENS^2 × UNITS + 3 *times* TENS × UNITS^2 + UNITS^3.

2. *The cube of a number consisting of three figures equals* HUNDREDS^3 + 3 *times* HUNDREDS^2 × TENS + 3 *times* HUNDREDS × TENS^2 + TENS^3 + 3 *times* (HUNDREDS + TENS)^2 × UNITS + 3 *times* (HUNDREDS + TENS) × UNITS^2 + UNITS^3.

553. These principles may also be expressed by letters, as follows:

$$(t+u)^3=t^3+3t^2.u+3t.u^2+u^3$$

$$(h+t+u)^3=h^3+3h^2.t+3h.t^2+t^3+3(h+t)^2.u + 3(h+t).u^2+u^3.$$

EVOLUTION.

554. **Evolution** is the process of finding a root of a number.

555. A **Root** of a number is *one* of its *equal* factors. Roots are of different degrees; as, *second*, *third*, etc.

556. The **Square Root**, or *second root*, of a number is *one* of its *two equal* factors. Thus, 8 is the square root of 64, since $8 \times 8 = 64$.

557. The **Cube Root**, or *third root*, of a number is *one* of its *three equal* factors. Thus, 4 is the cube root of 64, since $4 \times 4 \times 4 = 64$.

558. The **Fourth Root** is *one* of the *four equal* factors, the *fifth root* is *one* of the *five equal* factors, etc.

559. The **Symbol of Evolution** is $\sqrt{}$; thus, $\sqrt[2]{64}$, or $\sqrt{64}$, denotes the square root of 64; $\sqrt[3]{64}$ denotes the cube root of 64.

560. The **Index** of the root is a small figure placed in the angle of the symbol. The *index* indicates the degree of the root.

Roots are also indicated by the denominator of a fractional exponent, thus, $9^{\frac{1}{2}}$ denotes $\sqrt{9}$; $27^{\frac{1}{3}}$ denotes $\sqrt[3]{27}$, etc.

561. The following principles of involution are given to enable us to determine the number of figures in the root:

PRINCIPLES.

1. *The square of a number contains twice as many figures as the number itself, or twice as many less one.*

DEM.—The square of 1 is 1, and the square of 9 is 81; hence, the square of a number consisting of *one* figure is a number consisting of *one* or *two* figures. The square of 10, the smallest number of two figures, is 100; the square of 99, the largest number of two figures, is 9801; hence, the square of a number consisting of *two* figures is a number consisting of *three* or *four* figures—that is, *twice two*, or *twice two less one*, etc. The same may be shown for the square of a number consisting of any number of figures.

$1^2 = 1$

$9^2 = 81$

$10^2 = 100$

$99^2 = 9801$

2. *The cube of a number contains three times as many figures as the number itself, or three times as many, less one or two.*

DEM.—The cube of 1 is 1, and the cube of 9 is 729; hence, the cube of any number consisting of one figure is a number consisting of *one*, *two*, or *three* figures. The cube of 10 is 1000, a number of four figures; the cube of 99 is 970299, a number of six figures; hence, the cube of a number consisting of two figures contains *four*, *five*, or *six* figures—that is, *three times* two, or *three times* two *less one* or *two*. The same may be shown for the cube of a number consisting of any number of figures.

$1^3 = 1$

$9^3 = 729$

$10^3 = 1000$

$99^3 = 970299$

EVOLUTION BY FACTORING.

562. When the number is a perfect power and the factors are easily found, the root of a number can be readily obtained by the following

Rule.—*Resolve the number into its prime factors, and for the square root form a product by taking* ONE *of every* TWO *equal factors; for the cube root,* ONE *of every* THREE *equal factors,* etc.

WRITTEN EXERCISES.

1. Find the square root of 324.

SOLUTION.—We first resolve the number into its prime factors. Since the square root of a number is one of its two equal factors, we take *one* of every *two* equal factors and have $2 \times 3 \times 3 = 18$. Hence, the square root of 324 is 18.

NOTE.—We have marked the factors taken with a little star, and it will be well for the student to do the same in his solutions.

OPERATION.

2)324

*2)162

3)81

*3)27

3)9

*3

Solve the following problems:

2. $\sqrt{225}$.	*Ans.* 15.	**7.** $\sqrt[3]{19683}$.	*Ans.* 27.
3. $\sqrt{1024}$.	*Ans.* 32.	**8.** $\sqrt[3]{46656}$.	*Ans.* 36.
4. $\sqrt{4096}$.	*Ans.* 64.	**9.** $\sqrt[4]{5308416}$.	*Ans.* 48.
5. $\sqrt{9604}$.	*Ans.* 98.	**10.** $\sqrt[5]{7962624}$.	*Ans.* 24.
6. $\sqrt{11664}$.	*Ans.* 108.	**11.** $\sqrt[6]{11390625}$.	*Ans.* 15.

ORAL EXERCISES.

1. The square of a number is 64; what is the number?

2. The cube of a number is 27; what is the number?

3. The cube of a number is 125; what is the number?

4. Twice the square of a number is 98; what is the number?

5. Three times the square of a number is 108; what is the number?

6. One-third of the square of a number equals 48; what is the number?

7. The square of twice a number equals 100; what is the number?

8. The square of $\frac{2}{5}$ of a number equals 64; what is the number?

9. The square of twice a number is 18 more than twice the square of the number; what is the number?

10. Twice the square of a number is 8 more than 6 times the square of half the number; what is the number?

11. $\frac{2}{3}$ of the cube of a number is 10 more than the cube of $\frac{2}{3}$ of the number; what is the number?

12. The square of a number divided by the number equals 7; what is that number?

13. The cube of a number divided by the number equals 64; what is the number?

WRITTEN EXERCISES.

1. The third power of a number is 3375; what is the number? *Ans.* 15.

2. The fourth power of a number is 1296; what is the number? *Ans.* 6.

3. The product of the square and the cube of a number is 7776; what is the number? *Ans.* 6.

4. The product of the square of one number by the cube of another number is 675; what are the numbers?
Ans. 5 and 3.

SQUARE ROOT.

563. There are **Two Methods** of explaining the general process of extracting the square root, called the *Analytic* or *Algebraic Method*, and the *Synthetic* or *Geometrical Method*.

1. Teachers who prefer the geometrical method of explanation will find it in the Supplement.

2. With young pupils who have a difficulty in understanding evolution it may be well to drill them upon the method of doing the work, not requiring them to give the explanation until they are better prepared to understand it.

1. Extract the square root of 2025.

ANALYTICAL SOLUTION.—Since the square of a number *contains twice as many figures as the number itself, or twice as many less one*, the square root of 2025 will consist of two places, and hence will consist of tens and units, and 2025 consists of *tens*² + 2 × *tens* × *units* + *units*².

OPERATION.

$$\begin{array}{lrr} t^2 + 2t.u + u^2 = & 2025 & (40 \\ t^2 = 40^2 & 1600 & 5 \\ \hline 2t.u + u^2 = & 425 & \overline{45} \\ 2t = 40 \times 2 = 80 & & \\ (2t+u).u = (80+5) \times 5 = 425 & & \end{array}$$

The greatest number of tens whose square is contained in 2025 is 4 tens; squaring the tens and subtracting, we have 425, which equals 2 × *tens* × *units* + *units*². Now, since 2 × *tens* × *units* must be greater than *units*², 425 must consist principally of twice the tens into the units; hence, if we divide by *twice the tens*, we can ascertain the units. Twice the *tens* equals $40 \times 2 = 80$; dividing, we find the units to be 5; we now find 2 × *tens* × *units* + *units*², or, what is the same, 2 × *tens* + *units*, both multiplied by *units*, equals $(80+5) \times 5 = 425$; and subtracting, nothing remains. Hence, the square root of 2025 is 4 tens and 5 units, or 45.

NOTES.—1. When there are three figures in the root, we use the formula for three terms.

2. In practice we determine the number of figures in the root by pointing off the number into periods of two figures each, beginning at the right. We also abbreviate the work by omitting ciphers and condensing the other parts, preserving only the *trial* and *true* divisors. For illustration, see solution in the margin.

OPERATION.

$$\begin{array}{rr} & 10{\cdot}49{\cdot}76)324 \\ 3 & 9 \\ \hline 62 & 149 \\ & 124 \\ \hline 644 & 2576 \\ & 2576 \\ \hline \end{array}$$

Rule.—I. *Begin at units, and separate the number into periods of two figures each.*

II. *Find the greatest number whose square is contained in the left-hand period, place it at the right as a quotient, subtract its square from the left-hand period, and annex the next period to the remainder for a dividend.*

III. *Double the root found, and place it at the left for a* TRIAL DIVISOR; *divide the dividend, excluding the right-hand term, by this divisor for the second term of the root.*

IV. *Annex the second term of the root to the trial divisor for the* TRUE DIVISOR, *multiply the result by the second term of the root, subtract the product from the dividend, and bring down the next period for the next dividend.*

V. *Double the root now found for a second* TRIAL DIVISOR, *find the third term of the root as before, and thus proceed until all the periods have been used.*

1. If the product of a true divisor by a term of the root exceeds the dividend, the term of the root must be diminished by a unit.

2. When a cipher occurs in the root, annex a cipher to the trial divisor, bring down the next period, and proceed as before.

3. The square root of a common fraction is evidently the square root of each term. When these terms are not perfect squares, reduce the fraction to a decimal, and extract the root. When a number is not a perfect square, annex periods of ciphers and carry the root on to decimals.

4. By squaring 1, .1, .01, etc. we see that the *square of a decimal contains twice as many decimal places as the decimal;* hence, to extract the square root of a decimal we *point off the decimals into periods of two figures each, counting from the decimal point*, and proceed as in whole numbers.

$1^2 = 1$
$.1^2 = .01$
$.01^2 = .0001$

WRITTEN EXERCISES.

Extract the square root of

2. 289.	*Ans.* 17.	**9.** 59049.	*Ans.* 243.
3. 676.	*Ans.* 26.	**10.** 65536.	*Ans.* 256.
4. 784.	*Ans.* 28.	**11.** 164836.	*Ans.* 406.
5. 1369.	*Ans.* 37.	**12.** 277729.	*Ans.* 527.
6. 3136.	*Ans.* 56.	**13.** 390625.	*Ans.* 625.
7. 4096.	*Ans.* 64.	**14.** 5764801.	*Ans.* 2401.
8. 9216.	*Ans.* 96.	**15.** 7387524.	*Ans.* 2718.

Find the square root of

16. $\frac{169}{196}$.	*Ans.* $\frac{13}{14}$.	**24.** .064516.	*Ans.* .254.
17. $\frac{225}{256}$.	*Ans.* $\frac{15}{16}$.	**25.** 49.4209.	*Ans.* 7.03.
18. $\frac{784}{1089}$.	*Ans.* $\frac{28}{33}$.	**26.** 402.0025.	*Ans.* 20.05.
19. $\frac{1444}{2401}$.	*Ans.* $\frac{38}{49}$.	**27.** 2141.8384.	*Ans.* 46.28.
20. .2809.	*Ans.* .53.	**28.** 3.	*Ans.* 1.732+.
21. .6084.	*Ans.* .78.	**29.** $\frac{2}{3}$.	*Ans.* .81649+.
22. .0729.	*Ans.* .27.	**30.** $\frac{16}{111}$.	*Ans.* .37966+.
23. .059049.	*Ans.* .243.		

APPLICATIONS OF SQUARE ROOT.

564. The **Applications of Square Root** to problems involving geometrical figures are extensive.

Principle.—*The side of a square is equal to the square root of its area.*

WRITTEN EXERCISES.

1. A man owns a farm in the form of a square which contains 40 acres; how many rods in length or breadth is it?

SOLUTION.—The 40 acres equal 40 × 160, or 6400 sq. rd.; extracting the square root, we have 80 rods.

2. Henry has a square piece of land containing 640 acres; what is the length of one side? *Ans.* 320 rods.

3. How many rods of fence are required to enclose a square lot having an area of 9216 sq. ft.? *Ans.* $23\frac{3}{11}$ rods.

4. How many acres of land, in the form of a square, can be enclosed by 320 rods of fence? *Ans.* 40 acres.

5. What would it cost to fence a square lot containing 640 acres at $4 per rod? *Ans.* $5120.

6. A man has 300 yards of carpet $\frac{3}{4}$ yd. wide; what is the length of one side of a square room which the carpet will exactly cover? *Ans.* 15 yds.

7. A cabinet-maker has a board 15 ft. long by 1 ft. 8 in.

wide; what is the side of the largest square table he can make from it? *Ans.* 5 ft.

8. A general, wishing to form his army into a solid square of 75 men on a side, found he lacked 625 men to complete the square; how many men in his army? *Ans.* 5000.

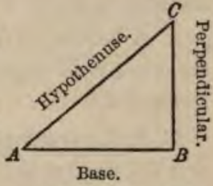
9. A general, attempting to form an army of 8430 men into a square, found that he had 149 men over; what was the number on a side of the square? *Ans.* 91.

RIGHT TRIANGLES.

565. A **Right Triangle** is a triangle which has one right angle.

566. The **Base** of a triangle is the side on which it stands; as, *AB*.

567. The **Perpendicular** is the side which forms the right angle with the base; as, *BC*.



568. The **Hypothenuse** is the side opposite the right angle; as, *AC*.

569. The **Principles** of right triangles, derived from Geometry, are as follows:

PRINCIPLES.

1. *The square of the hypothenuse equals the sum of the squares of the other two sides.*

2. *Hence, the square of either side equals the square of the hypothenuse, diminished by the square of the other side.*

NOTE.—The smallest integers which can express the relation of the three sides of a right triangle are 3, 4, and 5. We may have an infinite number of right triangles with their sides in this relation. Other relations are 5, 12, and 13; 8, 15, and 17, etc.

WRITTEN EXERCISES.

1. The two sides of a right triangle are 60 and 80 inches, respectively; required the hypothenuse.

SOLUTION.—Hypothenuse $=\sqrt{60^2+80^2}=\sqrt{10000}=100$, *Ans.*

2. Two rafters, each 34 feet long, meet at the ridge of a roof 16 feet above the top of the walls; what is the width of the house? *Ans.* 60.

3. A rope 85 ft. long stretches from the top of a derrick to a point on the ground 40 feet from the foot of the derrick; how high is the derrick? *Ans.* 75 ft.

4. A rectangular lot of land is 1260 rods long and 525 rods broad; what is the distance between two opposite corners? *Ans.* 1365 rods.

5. Two vessels sail from the same port: one sails north 9 miles an hour, the other sails west 12 miles an hour; how far are they apart in 2 days? *Ans.* 720 miles.

6. A ladder 26 ft. long stands close against a building; how far must it be drawn out at the bottom that the top may be lowered 2 feet? *Ans.* 10 ft.

7. A telegraph-pole was broken 12 feet from the bottom, and fell so that the end struck 16 ft. from the foot; required the length of the pole. *Ans.* 32 ft.

8. A ladder 130 ft. long, with its foot in the street, will reach on one side to a window 78 ft. high, and on the other to a window 50 ft. high; what is the width of the street? *Ans.* 224 ft.

SIMILAR FIGURES.

570. Similar Figures are those which have the same form. Thus, circles are similar figures; also squares, etc.

571. The **Principles** of similar figures, derived from Geometry, are as follows:

PRINCIPLES.

1. *The areas of all similar figures are to each other as the squares of their like dimensions.*

2. *Hence, the like dimensions of similar figures are to each other as the square roots of their areas.*

WRITTEN EXERCISES.

1. The area of a rectangle is 320, and one side is 16; what is the area of a similar rectangle, the shorter side being 24?

Solution.—Since the rectangles are similar, their areas are as the squares of their corresponding sides; hence, we have the proportion in the margin. Cancelling and multiplying, we have 720.

OPERATION.

Area of 2d : $320 = 24^2 : 16^2$

Area of 2d $= \frac{320 \times 24^2}{16^2} = 720$, *Ans.*

2. I have a lot 25 rd. long and 18 rd. broad; what are the dimensions of a similar lot 4 times as large? *Ans.* 50; 36.

3. There are two circular gardens, one 8 rods in diameter, and the other 32 rods; the second is how many times as large as the first? *Ans.* 16 times.

4. The area of a circle whose diameter is 10 feet is 78.54 square feet; what is the diameter of a circle whose area is 1256.64 square feet? *Ans.* 40 ft.

5. A farmer has a rectangular field 60 rods long and 45 wide; what are the dimensions of another similar field containing 30 acres? *Ans.* 80 rd.; 60 rd.

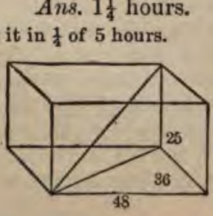
6. If a horse tied to a post in the centre of a field by a rope, 1 ch. $78\frac{1}{4}$ li., can graze upon an acre, what length of rope would allow it to graze upon $5\frac{4}{9}$ acres? *Ans.* 4 ch. $15\frac{1}{12}$ li.

7. If a pipe $\frac{3}{4}$ in. in diameter pours into a cistern 25 gal. in a given time, how much will a pipe 3 in. in diameter pour into the cistern in the same time? *Ans.* 400 gal.

8. If a pipe whose diameter is 1.5 in. fills a cistern in 5 hours, in what time will a pipe whose diameter is 3 in. fill the same cistern? *Ans.* $1\frac{1}{4}$ hours.

Sug.—It pours in four times as much, and fills it in $\frac{1}{4}$ of 5 hours.

9. Required the distance between a lower corner and the opposite upper corner of a room 48 feet long, 36 feet wide, and 25 feet high. *Ans.* 65 feet.



10. Two men, A and B, started from one corner of a square field containing 40 acres: A went directly across to the opposite corner, while B walked round the sides of the field; if A, after reaching the corner, goes along the side to meet B, how far from the corner would they meet? *Ans.* 23.431+ rods.

CUBE ROOT.

572. There are **Two Methods** of explaining the general process of extracting the cube root, called the *Analytic* or *Algebraic Method* and the *Synthetic* or *Geometrical Method.*

Teachers who prefer the geometrical method of explanation will find it in the Supplement.

1. Extract the cube root of 97336.

OPERATION.

$$\begin{array}{rr|ll}
 & & 97336 & (40 \\
 & 40^3 = & \underline{64000} & \underline{6} \\
\textit{trial divisor, } 3\times 40^2 & = 4800 & 33336 & 46 \\
3\times 40\times 6 & = 720 & & \\
6^2 & = \underline{36} & & \\
\textit{true divisor,} & 5556 & 33336 &
\end{array}$$

SHOWN BY LETTERS.

$$\begin{array}{l}
t^3+3t^2\times u+3t\times u^2+u^3=97336)46 \\
\underline{t^3=40^3 \qquad\qquad\qquad\quad =64000} \\
\quad 3t^2\times u+3t\times u^2+u^3=33336 \\
3t^2\times 3\times 40^2 \qquad\quad =4800 \\
3t\times u=3\times 40\times 5= 720 \\
\quad \underline{u^2=5^2 \qquad\qquad\quad 36} \\
(3t^2+3t\times u+u^2)\times u=5556\times 6=33336
\end{array}$$

ANALYTIC SOLUTION.—Since the cube of a number consists of *three times as many places as the number* itself, or of *three times as many less one or two*, the cube root of 97336 will consist of two places, or of tens and units, and the number itself will consist of $tens^3+3\times tens^2\times units+3\times tens\times units^2+units^3$.

The greatest number of tens whose cube is contained in 97336 is 4 tens. Cubing the tens and subtracting, we have 33336, which equals $3\times tens^2\times units+3\times tens\times units^2+units^3$. Now, since $3\times tens^2\times units$ is much greater than $3\times tens\times units^2+units^3$, 33336 consists principally of 3 times $tens^2\times units$; hence, if we divide by 3 times $tens^2$, we can ascertain the *units*: 3 times $tens^2$ equals $3\times 40^2=4800$; dividing by 4800, we find the *units* to be 6. We then find 3 times $tens\times units$ equal to $3\times 40\times 6=720$, and $units^2=6^2=36$, and, adding these and multiplying by *units*, we have $(3\ tens^2+3\ tens\times units+units^2)\times units$, which equals $5556\times 6=33336$; subtracting, nothing remains; hence, the cube root of 97336 is 46.

2. Find the cube root of 14348907.

We will solve this problem, indicating the solution by means of letters, and abbreviating the operation as in practice. A point like a period indicates the multiplication of the letters.

SHOWN BY LETTERS.

$$
\begin{array}{rr|l}
\multicolumn{3}{r}{htu} \\
\multicolumn{3}{r}{14348907(243} \\
h^3 = 200^3 = & \multicolumn{2}{l}{8000000} \\
\cline{2-3}
 & & 6348907 \\
3h^2 = 3 \times 200^2 = & 120000 & \\
3h.t = 3 \times 200 \times 40 = & 24000 & \\
t^2 = 40^2 = & 1600 & \\
\cline{2-3}
 & 145600 & 5824000 \\
\cline{2-3}
 & & 524907 \\
3(h+t)^2 = 3 \times 240^2 = & 172800 & \\
3(h+t)u = 3 \times 240 \times 3 = & 2160 & \\
u^2 = 3^2 = & 9 & \\
\cline{2-3}
 & 174969 & 524907 \\
\cline{2-3}
\end{array}
$$

OPERATION AS IN PRACTICE.

$$
\begin{array}{rr|l}
\multicolumn{3}{r}{14{\cdot}348{\cdot}907(243} \\
2^3 = & 8 & \\
\cline{3-3}
 & & 6348 \\
2^2 \times 300 = & 1200 & \\
2 \times 4 \times 30 = & 240 & \\
4^2 = & 16 & \\
\cline{2-2}
 & 1456 & 5824 \\
\hline
 & & 524907 \\
24^2 \times 300 = & 172800 & \\
24 \times 3 \times 30 = & 2160 & \\
3^2 = & 9 & \\
\cline{2-2}
 & 174969 & 524907 \\
\cline{2-3}
\end{array}
$$

The method employed in actual practice is derived from the other by omitting ciphers, using parts of the number instead of the whole number each time we obtain a figure of the root, etc. It will also be seen that by separating the number into *periods of 3 figures each,* we have the *number of places in the root,* the *part of the number* used in obtaining each figure of the root, etc.

Rule.—I. *Begin at units and separate the number into periods of three figures each.*

II. *Find the greatest number whose cube is contained in the left-hand period, write it for the first term of the root, subtract its cube from the left-hand period, and annex the next period to this remainder for a dividend.*

III. *Square the first term of the root and multiply by* 300 *for a* TRIAL DIVISOR; *divide the dividend by the trial divisor for the second term of the root.*

IV. *To the trial divisor add* 30 *times the product of the second term of the root by the first term, and also the square of the second term; their sum will be the* TRUE DIVISOR.

V. *Multiply the true divisor by the second term of the root, subtract the product from the dividend, and annex the next period for another dividend.*

VI. *Square the root now found, multiply by 300, and find the third term of the root as before, and thus continue until all the periods have been used.*

1. If the product of the true divisor by the term of the root exceeds the dividend, the root must be diminished by a unit.

2. When a dividend will not contain a trial divisor, place a cipher in the root and two ciphers at the right of the trial divisor; bring down the next period, and proceed as before.

3. To find the cube root of a common fraction, extract the cube root of both terms. When these are not perfect cubes, reduce to a decimal and then extract the root.

4. By cubing 1, .1, .01, etc. we see that the *cube of a decimal contains three times as many decimal places as the decimal;* hence, to extract the cube root of a decimal we *point off the decimal into periods of three figures each, counting from the decimal point.*

$1^3=1$
$.1^3=.001$
$.01^3=.000001$

WRITTEN EXERCISES.

Find the cube root of

1. 12167.	*Ans.* 23.
2. 32768.	*Ans.* 32.
3. 39304.	*Ans.* 34.
4. 46656.	*Ans.* 36.
5. 68921.	*Ans.* 41.
6. 148877.	*Ans.* 53.
7. 274625.	*Ans.* 65.
8. 438976.	*Ans.* 76.
9. 551368.	*Ans.* 82.
10. 614125.	*Ans.* 85.
11. 2571353.	*Ans.* 137.
12. 12326391.	*Ans.* 231.
13. 34965783.	*Ans.* 327.
14. 1124864.	*Ans.* 104.
15. 8998912.	*Ans.* 208.
16. 41063625.	*Ans.* 345.
17. 43614208.	*Ans.* 352.
18. 75686967.	*Ans.* 423.
19. $\frac{2197}{9261}$.	*Ans.* $\frac{13}{21}$.
20. 185.193.	*Ans.* 5.7.
21. 629.422793.	*Ans.* 8.57.
22. 4.	*Ans.* 1.5874+.
23. 5.	*Ans.* 1.7099+.
24. 6.	*Ans.* 1.8171+.
25. $\frac{2}{3}$.	*Ans.* .873+.
26. 1879080904.	*Ans.* 1234.

NOTE.—For a shorter method of Cube Root, see Supplement.

APPLICATIONS OF CUBE ROOT.

573. The **Applications** of cube root to problems involving geometrical volumes, such as cubes, parallelopipeds, spheres, etc., are extensive.

Principle.—*The edge of a cube is equal to the cube root of the contents of the cube.*

WRITTEN EXERCISES.

1. Required the dimensions of a cubical chest which shall contain 125000 cubic feet. *Ans.* 50 ft.

2. The pedestal of a certain statue is a cubical block whose contents are 175616 cu. ft.; required the number of square feet in one face. *Ans.* 3136.

3. What is the entire surface of a cube whose cubical contents are 74088 cubic feet? *Ans.* 10584 sq. ft.

4. What is the edge of a cube which shall contain as much as a solid 30 ft. 4 in. long, 15 ft. 6 in. wide, and 9 ft. 8 in. high? *Ans.* 16.56 ft.+.

5. What is the depth of a cubical cistern which shall contain 300 gal. (231 cu. in.) of water? *Ans.* 41.07 in.+.

6. A farmer had a cubical bin which contained 60 bushels of grain; what was its depth? *Ans.* 4.21 ft.+.

7. What would it cost to plaster the bottom and sides of a cubical reservoir which contains 100 barrels (31½ gals.) of water at 6 cents a square foot? *Ans.* $16.85.

SIMILAR VOLUMES.

574. **Similar Volumes** are such as have the same shape, but differ in size; as, cubes, spheres, etc.

575. A **Dimension** of a volume is a length, breadth, height, diameter, radius, circumference, etc.

576. The **Principles** of similar volumes are derived from geometry.

PRINCIPLES.

1. *Similar volumes are to each other as the cubes of their like dimensions.*

2. *Like dimensions of similar volumes are to each other as the cube roots of those volumes.*

WRITTEN EXERCISES.

1. A man has two balls, one 8 in. in diameter, the other 4 in.; the first is how many times as large as the second?

SOLUTION.—By the principle above we have the proportion 1st : 2d : : 8^3 : 4^3; and since the 1st term equals the 2d term multiplied by the ratio of the 3d to the 4th, we have 1st term = 2d term multiplied by the ratio of 8^3 to 4^3, which is 2d term × $(\frac{8}{4})^3$, or 2d × 2^3, or 2d × 8. Hence, the 1st term is 8 times as large as the 2d.

OPERATION.

$1 : 2d : : 8^3 : 4^3$

$1st = 2d \times (\frac{8}{4})^3 = 2d \times 2^3$

$1st = 2d \times 8$

2. Required the relation of two cubes whose dimensions are 4 in. and 16 in. respectively. *Ans.* 2d is 64 times 1st.

3. If a ball 4 in. in diameter weigh 20 pounds, what will be the weight of a ball 6 in. in diameter? *Ans.* 67½ lb.

4. I have two cubical boxes, one having a length of 9 inches and the other of 12; how many times will the larger contain the smaller? *Ans.* $2\frac{10}{27}$ times.

5. If a haystack 10 ft. in diameter contain 12 tons of hay, what will be the contents of a stack of hay whose diameter is 15 feet? *Ans.* 40½ tons.

6. If a man 5 ft. 4 in. high weighs 160 lb., what is the weight of a man of similar build whose height is 6 ft.? *Ans.* $227\frac{13}{16}$ lb.

7. The sun is 885680 miles in diameter, and the earth 7912 miles; the sun is how many times as large as the earth? *Ans.* About 112^3.

8. There are two balls whose diameters are respectively 4 in. and 5 in.; what is the diameter of a ball whose contents are equal to the contents of both? *Ans.* 5.74 in.—.

SUG.—Cube 4 and 5, take their sum, and then compare this with either of the given balls.

HIGHER ROOTS.

577. Any root whose index contains only the factors 2 or 3 can be extracted by means of the square and cube root, according to the following principle:

PRINCIPLE.

A root of a number equals a root of a root of the number in which the product of the indices of the two latter roots equals the index of the former.

Since the square of the cube of a number equals the sixth power, the sixth root of a number equals the square root of the cube root of the number; and the same is true in any other case.

WRITTEN EXERCISES.

1. Extract the sixth root of 4096.

Solution.—To find the sixth root of 4096, we first extract the square root, which we find to be 64, and then find the cube root of 64, which is 4. Hence, the sixth root of 4096 is 4.

Required the value of

2. $\sqrt[4]{625}$. *Ans.* 5.

3. $\sqrt[6]{729}$. *Ans.* 3.

4. $\sqrt[4]{20736}$. *Ans.* 12.

5. $\sqrt[8]{390625}$. *Ans.* 5.

6. $\sqrt[9]{262144}$. *Ans.* 4.

7. $\sqrt[12]{16777216}$. *Ans.* 4.

Note.—For a general rule of Evolution, see *Brooks's Higher Arithmetic.*

MISCELLANEOUS EXERCISES.

1. The square of the square of a number equals 50625; what is the number? *Ans.* 15.

2. The square of the cube of a number equals 46656; what is the number? *Ans.* 6.

3. The square of a number multiplied by the cube of the number equals 4084101; what is the number? *Ans.* 21.

SECTION XI.

MENSURATION.

578. **Mensuration** treats of the measurement of geometrical magnitudes.

579. **Geometrical Magnitudes** consist of the *Line*, *Surface*, *Volume*, and *Angle*.

Teachers will illustrate the rules of Mensuration concretely, so far as possible.

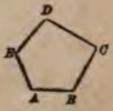
MENSURATION OF SURFACES.

580. A **Surface** is that which has length and breadth without thickness. Surfaces are *plane* or *curved*.

581. A **Plane Surface** is a surface such that if any two of its points be joined by a straight line, every part of that line will lie in the surface.

582. A **Plane Figure** is a plane surface bounded by lines either straight or curved.

583. A **Polygon** is a plane figure bounded by straight lines; as, *ABCDE*.



A Polygon of three sides is called a *Triangle;* of four sides, a *Quadrilateral;* of five sides, a *Pentagon;* of six sides, a *Hexagon*, etc.

584. A **Diagonal** of a polygon is a line joining the vertices of two angles not consecutive.

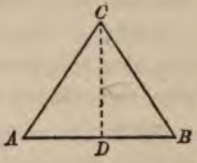
585. The **Perimeter** of a polygon is the sum of its sides.

586. The **Area** of a plane figure is the number of square units in its surface.

NOTE.—The principles of Mensuration are derived from geometry; their application to practical purposes is usually given in arithmetic.

THE TRIANGLE.

587. A **Triangle** is a polygon of three sides and three angles; as, *ABC*.



588. The **Base** is the side upon which it seems to stand; as, *AB*. The *Altitude* is a line perpendicular to the base, drawn from the angle opposite; as, *CD*.

589. An **Equilateral Triangle** is a triangle which has its three sides equal; when two sides are equal it is called *isosceles;* when its sides are unequal it is called *scalene*.

Rule.—*To find the area of a triangle, multiply the base by one-half of the altitude.*

NOTE.—If the three sides are given and not the altitude, *take half the sum of the sides, subtract from it each side separately, multiply the half sum and these remainders together, and take the square root of the product.*

WRITTEN EXERCISES.

1. Required the area of a triangle whose base is 45 rods and altitude 27 rods. *Ans.* 3 A. 127½ P.

2. Required the area of a triangular tract whose base is 645 rods and altitude 484 rods. *Ans.* 975 A. 90 P.

3. Required the area of an equilateral triangle whose sides are each 10 chains. *Ans.* 43.30+ ch.

4. What is the area of a field whose sides are respectively 30, 40, and 50 chains? *Ans.* 60 A.

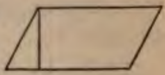
5. The rafters of a house are 25 ft. long on one side and 20 on the other, and the width of the house is 35 ft.; how many square feet of boards will it require to board up both gables? *Ans.* 489.9 sq. ft.

THE QUADRILATERAL.

590. A **Quadrilateral** is a polygon having four sides and four angles. There are three classes—the *parallelogram*, *trapezoid*, and *trapezium*.

591. A **Parallelogram** is a quadrilateral whose opposite sides are parallel. The *altitude* is the perpendicular distance between its opposite sides.

592. A parallelogram which is right-angled is called a *Rectangle.* When the four sides are equal it is called a *Square.*



593. An oblique-angled parallelogram is called a *Rhomboid.* An equilateral rhomboid is called a *Rhombus.*

Rule.—*To find the area of a parallelogram, multiply the base by the altitude.*

WRITTEN EXERCISES.

1. What is the area of a parallelogram 30 feet long and 24 feet wide? *Ans.* 80 sq. yd.

2. A has a rectangular lot 180 chains long and 60 chains wide; what is its area? *Ans.* 1080 A.

3. What is the difference in the area of two lots, one being 354 rd. long, 52 rd. wide, and the other 95 chains long and 28 chains wide? *Ans.* 150 A. 152 P.

4. A carpenter had a plank 18 inches wide, from which he wished to saw off 9 square feet; what will be the length of the piece sawed off? *Ans.* 6 ft.

594. A **Trapezoid** is a quadrilateral which has two of its sides parallel. Its *altitude* is the perpendicular distance between its parallel sides.

Rule.— *To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

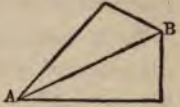
WRITTEN EXERCISES.

1. Required the area of a trapezoid, one side being 150 in., the other 90 in., and the altitude 48 in. *Ans.* 40 sq. ft.

2. What is the area of a plank 15 feet long, 20 inches wide at one end and 12 inches at the other end? *Ans.* 20 sq. ft.

3. A farm in the form of a trapezoid has its parallel sides 72 ch. and 84 ch. in length, and the perpendicular distance between them is 40 ch.; how large is the farm? *Ans.* 312 A.

4. A farmer has two fields, one in the form of a trapezoid, the two parallel sides being 40 and 44 rods respectively, and the perpendicular distance between them being 28 rods; and the other a rectangle, 42 rods long and 28 rods wide; what is the difference in their areas? *Ans.* They are equal.



595. A **Trapezium** is a quadrilateral which has none of its sides parallel. A diagonal, as AB, divides the trapezium into two triangles.

Rule.—*To find the area of a trapezium, divide the trapezium into two triangles by a diagonal, find the area of each triangle, and take the sum.*

WRITTEN EXERCISES.

1. What is the area of a trapezium whose diagonal is 145 in., and the altitudes of the triangles, the diagonal being the base, are 32 and 48 in. respectively? *Ans.* 40 sq. ft. 40 sq. in.

2. Required the area of a farm in the form of a trapezium the length of whose sides are respectively 30, 40, 35, and 45 chains, and the length of the diagonal 50 chains.

Ans. 136 A. 77.6+ P..

THE CIRCLE.

596. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *centre.*

597. The curved line is called the *circumference,* and a line passing through the centre and ending in the circumference is the *diameter.* Half the diameter is called the *radius.*

Rule.—I. *The circumference of a circle equals the diameter multiplied by* 3.1416.

II. *The diameter of a circle equals the circumference multiplied by* .3183.

WRITTEN EXERCISES.

1. What is the circumference of a wheel whose diameter is 5 feet 8 inches? *Ans.* 17.8024 ft.

2. What is the distance around a circular fish-pond, the diameter of which is 20 rods? *Ans.* 62.832 rd.

3. A man has a garden in the form of a circle, the diameter of which is 28 rods; what is the distance around the garden? *Ans.* 87.9648 rd.

4. What is the diameter of a water-wheel whose circumference is 47.124 feet? *Ans.* 15 feet.

5. What is the diameter of a circular park whose circumference is 2 miles? *Ans.* 203.712 rd.

Rule.—I. *The area of a circle equals the circumference multiplied by one-fourth of the diameter, or the square of the circumference multiplied by* .07958.

II. *The area of a circle equals the square of the radius multiplied by* 3.1416, *or the square of the diameter multiplied by* .785398.

III. *To find the diameter of a circle when the area is given, divide the area by* .785398, *and extract the square root.*

NOTE.—The area will vary slightly in the decimal figures as we use the different rules.

WRITTEN EXERCISES.

1. What is the area of a circle whose diameter is 15 and circumference 47.124? *Ans.* 176.715.

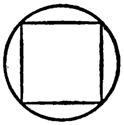
2. What is the area of a circular park whose diameter is 210 rods? *Ans.* 216 A. 76.0518 P.

3. What is the area of a circular garden whose circumference is 150 rods? *Ans.* 11 A. 30.55 P.

4. The area of a circle is 176.715; what are the diameter and the circumference? *Ans.* Diam., 15; circ., 47.124.

5. If a cow is fastened to a post in a field, how long must the rope be to allow her to graze over 1 acre? *Ans.* 7.136+ rd.

6. A circular park contains 120 acres; how many rods of fence will be needed to go round it? *Ans.* 491.189 rd.



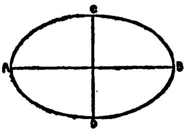
598. A square is inscribed in a circle when each of its angles is in the circumference.

Rule.—*To find the side of an inscribed square, multiply the diameter by* .707106 *or multiply the circumference by* .225079.

1. The diameter of a round stick of timber is 2 ft. 6 in.; what is the side of a square stick that can be cut out of it? *Ans.* 1.767 + ft.

2. How large a square can be cut out of a circular board whose circumference is 250 inches? *Ans.* 56.26975 in.

THE ELLIPSE.



599. An **Ellipse** is a plane figure bounded by a curved line, the sum of the distances from every point of which to two fixed points is equal to the line drawn through these points and terminated by the curve. The two fixed points are called *foci;* the line through the foci is the *transverse* axis; and a line perpendicular to this passing through the centre and terminated by the curve is the *conjugate* axis.

Rule.—*To find the area of an ellipse, multiply half of the two axes together, and that product by* 3.1416.

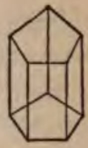
1. What is the area of an ellipse whose transverse axis is 50 inches and conjugate axis is 18 inches? *Ans.* 706.86 sq. in.

2. Required the area of an elliptical mirror whose length is 5.5 feet and breadth 4 feet. *Ans.* 17.2788 sq. ft.

MENSURATION OF VOLUMES.

600. A **Volume** is that which has length, breadth, and thickness.

THE PRISM.



601. A **Prism** is a volume whose ends are equal and parallel polygons and whose sides are parallelograms.

602. The polygons are called *bases*, the parallelograms form the *lateral surface*, and the prism takes its name from the form of its base.

603. The **Parallelopiped** is a prism whose bases are parallelograms. A *cube* is a parallelopiped all of whose sides are squares.

Rule.—I. *The lateral surface of a prism equals the perimeter of the base multiplied by the height.*

NOTE.—To find the entire surface we add the area of the bases.

II. *The contents of a prism equal the area of the base multiplied by the altitude of the prism.*

WRITTEN EXERCISES.

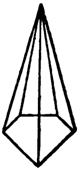
1. What is the lateral surface of a triangular prism, the three sides of whose base are respectively 6, 7, and 8 feet, and height 50 feet? *Ans.* 1050 sq. ft.

2. What is the entire surface of the triangular prism given in the first problem? *Ans.* 1090.66 sq. ft.

3. What are the contents of a square prism whose altitude is 40 feet and the side of the base 4 feet? *Ans.* 640 cu. ft.

4. Required the contents of a triangular prism, the sides of whose base are each 20 inches, and whose altitude is 30 inches. *Ans.* 5196.15 cu. in.

THE PYRAMID.



604. The **Pyramid** is a volume bounded by a polygon and several triangles meeting in a common point. The polygon is called the *base*, and the triangles form the *lateral surface.*

605. The point at the top is called the *vertex;* the distance from the vertex to the base is the *altitude;* the distance from the vertex to the middle of a side is the *slant height.*

Rule.—I. *The lateral surface of a pyramid equals the perimeter of the base multiplied by one-half the slant height.*

II. *The contents of a pyramid equal the area of the base multiplied by one-third of the altitude.*

WRITTEN EXERCISES.

1. What is the lateral surface of a triangular pyramid, the sides of whose base are each 5 ft., and whose slant height is 30 ft.? *Ans.* 225 sq. ft.

2. Required the lateral surface of a pentagonal pyramid, the sides of whose base are each 6 ft., and whose slant height is 50 ft. *Ans.* 750 sq. ft.

3. What will be the cost of painting a hexagonal church-spire at 30¢ a sq. yd., the sides of the base being 6 ft. and slant height 75 ft.? *Ans.* $45.

4. Required the contents of a pyramid whose base is 6 ft. square, and whose altitude is 75 ft. *Ans.* 900 cu. ft.

5. Required the contents of a pyramid whose base is a triangle, each side of which is 10 ft., and the altitude of the pyramid is 84 ft. *Ans.* 1212.428 cu. ft.

6. What is the weight of the pyramid in Ex. 5 if it is composed of granite weighing 165 lb. a cubic foot?

An

THE CYLINDER.

606. The **Cylinder** is a body bounded by a uniformly curved surface of uniform diameter, with circles for its ends. The two circular ends are called *bases.*

607. The **Altitude** of a cylinder is the distance from the centre of one base to the centre of the other.

Rule.—I. *To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.*

II. *The contents of a cylinder equal the area of the base multiplied by the altitude.*

WRITTEN EXERCISES.

1. What is the convex surface of a cylinder, altitude 10 ft. and diameter of base 5 ft.? *Ans.* 157.08 sq. ft.

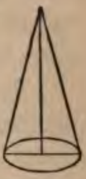
2. What is the convex surface of a cylinder 50 feet long and 16 feet in diameter? *Ans.* 2513.28 sq. ft.

3. Required the contents of a cylinder 40 feet long and 6 feet in diameter. *Ans.* 1130.976 cu. ft.

4. Required the contents of a cylindrical log 15 feet long and $8\frac{2}{3}$ feet in diameter. *Ans.* 884.884 cu. ft.

THE CONE.

608. A **Cone** is a volume whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex.*



609. The **Altitude** of a cone is the distance from the vertex to the centre of the base, and the *slant height* is the distance from the vertex to the circumference of the base.

Rule.—I. *The convex surface of a cone equals the circumference of the base multiplied by one-half the slant height.*

II. *The contents of a cone equal the area of the base multiplied by one-third of the altitude.*

WRITTEN EXERCISES.

1. What is the convex surface of a cone, the circumference of whose base is 72 inches and slant height 60 inches? *Ans.* 2160 sq. in.

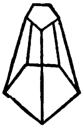
2. I have a conical haystack whose slant height is 9.25 ft., and the diameter of the base is 8.5 ft.; how many square yards of canvas will cover it completely? *Ans.* 13.723 sq. yd.

3. Required the contents of a sugar-loaf, diameter of the base being 6 in. and height 15 in. *Ans.* 141.372 cu. in.

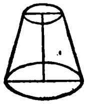
4. How many cubic feet in a conical haystack 8 ft. high, and base 24 ft. in circumference? *Ans.* 122.227 cu. ft.

THE FRUSTUM OF A PYRAMID AND CONE.

610. The **Frustum of a Pyramid** is the part of a pyramid which remains after cutting off the top by a plane parallel to the base.



611. The **Frustum of a Cone** is the part of a cone which remains after cutting off the top by a plane parallel to the base.



Rule.—I. *The lateral or convex surface of a frustum equals the sum of the perimeters or circumferences of the two bases multiplied by one-half the slant height.*

II. *To find the contents of a frustum, take the sum of the two bases and the square root of their product, and multiply this sum by one-third of the altitude of the frustum.*

WRITTEN EXERCISES.

1. Required the lateral surface of the frustum of a square pyramid whose slant height is 20 feet, the side of the lower base 18 feet and upper base 12 feet. *Ans.* 1200 sq. ft.

2. Required the surface of a frustum of a cone whose slant height is 12 feet, diameter of lower base 10 feet and upper base 6 feet. *Ans.* 301.5936 sq. ft.

3. Required the cost, at 12½¢ a sq. yd., of painting the lateral surface of a vat 12 ft. in diameter at the bottom and 10 ft. at the top, the slant height being 14 ft. *Ans.* $6.72.

4. What are the contents of the frustum of a square pyramid the sides of whose bases are 12 and 18 feet, and whose altitude is 18 feet? *Ans.* 4104 cu. ft.

Sug.—$12^2+18^2+\sqrt{12^2\times18^2}=144+324+216=684$, and this multiplied by 6 equals 4104 cu. ft.

5. What is the value of the wheat, at $0.75 a bushel, contained in a bin the lower base of which is 8 ft. square, the upper base 12 ft. square, and the height 9 ft.? *Ans.* $547.20.

6. What is the amount of timber in a log which measures 60 feet in length, the radius of one base being 8 feet and of the other 5 feet? *Ans.* 8105.328 cu. ft.

7. A vat measures 12 feet in diameter at top, 15 feet at bottom, and is 10 feet deep; how many gallons of water will be required to fill it? *Ans.* 10751.616 + gal.

THE SPHERE.

612. A **Sphere** is a volume bounded by a curved surface, every point of which is equally distant from a point within called the *centre.*

613. The **Diameter** of a sphere is a line passing through its centre and ending in the surface. The *radius* is half the diameter.

Rule.—I. *The surface of a sphere equals the circumference multiplied by the diameter, or the square of the radius multiplied by* 4 *times* 3.1416.

II. *The contents of a sphere equal the cube of the diameter multiplied by* $\frac{1}{6}$ *of* 3.1416.

III. *To find the edge of a cube which may be cut from a given sphere, square the diameter, divide by 3, and extract the square root of the quotient.*

WRITTEN EXERCISES.

1. Required the surface of a sphere whose diameter is 36 inches. *Ans.* 4071.5136 sq. in.

2. Required the surface of a sphere whose diameter is 108 inches. *Ans.* 36643.6224 sq. in.

3. Required the contents of a cannon-ball whose diameter is 9 inches. *Ans.* 381.7044 cu. in.

4. The diameter of the earth is 8000 miles; required its surface and solid contents. *Ans.* Sur., 201062400 sq. mi.

5. What is the weight of a cannon-ball 20 in. in diameter, a cu. ft. of iron weighing about 450 lb. *Ans.* 1090.83+ lb.

6. How many cubic feet of iron in a hollow shell whose outside diameter is 16 inches and inside diameter 12 inches? *Ans.* .717+ cu. ft.

7. What is the edge of a cube which may be cut from a sphere 24 inches in diameter? *Ans.* 13.856+ in.

8. What is the edge of a cube which can be cut from a billiard-ball whose circumference is 9½ inches? *Ans.* 1.746— in.

GAUGING.

614. **Gauging** is the process of finding the capacity of casks and other vessels.

Barrels and casks differ from cylinders in bulging out in the middle. By ascertaining the approximate mean diameter of the cask or barrel, the capacity can be obtained like that of a cylinder.

Rule.—I. *To find the mean diameter of a barrel or cask, add to the head diameter* $\frac{2}{3}$, *or, if the staves are not much curved,* $\frac{3}{5}$, *of the difference between the head and bung diameters.*

II. *To find the capacity in gallons, multiply the square of the mean diameter by the length (both expressed in inches), and this product by* .0034.

WRITTEN EXERCISES.

1. How many gallons in a cask whose head diameter is 28 in., bung diameter 36 in., and length 40 in.? *Ans.* $151\frac{1}{9}$ gal.

2. How many gallons in a barrel of slight curvature, 3 ft. long, the head diameter being 26 in. and the bung diameter 29 in.? *Ans.* 94.595616 gal.

EXERCISES IN MENSURATION.

1. Two towns, 42 miles apart, are on a map located $10\frac{1}{2}$ inches apart; what is the scale on which the map is drawn? *Ans.* $\frac{1}{4}$ in. to the mi.

2. How many feet of boards will cover the gable end of a house 34 ft. wide, the ridge being 18 ft. high? *Ans.* 306 sq. ft.

3. The rafters of a roof are 18 ft. long, and the distance between the eaves is 24 ft.; what is the height of the ridge? *Ans.* 13.41+ ft.

4. I have a triangular building lot whose sides measure 25, 35, and 40 feet respectively; if I sell it at $5 per square foot, what do I receive? *Ans.* $2165.05.

5. How many dozen boxes of perfumery, 2 in. on each side, can be packed in a rectangular box whose dimensions are respectively 1 ft. 6 in., 1 ft., and 8 in.? *Ans.* 18 doz.

6. A cubic inch of gold is hammered out to cover a *square* (100 sq. ft.); what is its thickness? *Ans.* .000069 inch.

7. The cost of a cube of metal, at $12 per cu. in., is $4116; find the cost of gilding it over at 2¢ per sq. in. *Ans.* $5.88.

8. If 50 cubic feet of air are required per person in a well-ventilated room, how many persons can safely remain in a room 60 ft. long, 40 ft. wide, and 12 ft. high? *Ans.* 576 persons.

MISCELLANEOUS EXAMPLES.

FUNDAMENTAL RULES.

1. If 256 is added to the difference of two numbers, the result will be 368; if the smaller number is 729, what is the larger number? *Ans.* 841.

2. If 441 is added to the difference of two numbers, the result will be 657; if the larger number is 540, what is the smaller number? *Ans.* 324.

3. If we subtract a number twice from 1876, the sum of the two results will be 2252; what is the subtracted number? *Ans.* 750.

4. If a number is subtracted from 2085, and then subtracted from the remainder, the result will be 335; what is the subtracted number? *Ans.* 875.

5. A number of marbles divided among 29 boys gives each boy 16 marbles, and leaves a remainder of 26 marbles; how many marbles are there? *Ans.* 490.

6. If three times a number is subtracted from 4236, the remainder will be 2316; what is the number? *Ans.* 640.

7. The product of two numbers is 45260; if one number is 365, what is the other number? *Ans.* 124.

8. If the product of two numbers is increased by 625, the result will be 81625; if one number is 324, what is the other? *Ans.* 250.

9. The product of two numbers, divided by 81, is 700; if one number is 350, what is the other? *Ans.* 162.

10. The product of three numbers is 746496, and the product of the two smaller numbers is 6912; if one of the smaller numbers is 96, what are the others? *Ans.* 72 and 108.

11. The quotient of two numbers, increased by 25, is 97; if the smaller number is 108, what is the larger? *Ans.* 7776.

12. The quotient of two numbers, diminished by 16, is 80; if the smaller number is 72, what is the larger? *Ans.* 6912.

13. The quotient of two numbers, multiplied by the smaller number, is 5120; if the smaller number is 80, what is the larger? *Ans.* 5120.

14. The quotient of two numbers, divided by the smaller number, is 3; if the smaller number is 48, what is the larger? *Ans.* 6912.

15. If two numbers are each divided by 3, the product of these results will be 1215; if one number is 81, what is the other? *Ans.* 135.

16. If two numbers are each divided by 4, the quotient of these results will be 3; if one number is 96, what is the other? *Ans.* 288.

17. If one number is multiplied by 3 and another divided by 4, the product of these results would be 1440; if one number is 24, what is the other? *Ans.* 80.

18. If one number is multiplied by 4 and the other divided by 3, the quotient of the results will be 3; if one number is 25, what is the other? *Ans.* 900.

19. At the rate of 30 problems in an hour for A, and 15 in 45 minutes for B, in what time can they together solve 100 problems? *Ans.* 2 hours.

20. If a boy that steps 28 inches takes 1000 steps in going to school, how many steps will his brother take, whose step is 35 inches? *Ans.* 800 steps.

21. If a number is used three times as a factor, the result will be 1157625; what is the number? *Ans.* 105.

22. If a number is used four times as a factor, the result will be 3111696; what is the number? *Ans.* 42.

23. Find the greatest common divisor and the least common multiple of 36, 48, 60, and 72. *Ans.* 12; 720.

24. Find the greatest common divisor and least common multiple of 847, 1331, and 1573? *Ans.* 121; 121121.

Find the greatest common divisor of

25. 1220 and 2013. *Ans.* 61.

26. 1241 and 1460. *Ans.* 73.

27. 9301 and 13231. *Ans.* 131.

28. 72491 and 103121. *Ans.* 1021.

Find the least common multiple of

29. 357 and 612. *Ans.* 4284.

30. 612 and 663. *Ans.* 7956.

31. 4141 and 6161. *Ans.* 252601.

32. 6161 and 7171. *Ans.* 437431.

33. What is the greatest length of boards I can use in fencing my garden, the sides of which are 168 feet, 182 feet, 280 feet, and 252 feet? *Ans.* 14 ft.

34. Mr. Bainbridge has a field whose sides measure 256, 292, 384, and 400 feet respectively; what is the length of the rails used to fence it if they are all of equal length and the longest that can be used? *Ans.* 4 ft.

35. A farmer makes 675 gallons of cider, and has but 12 barrels, each holding 45 gallons, to store it in; how many more such barrels does he need? *Ans.* 3 barrels.

36. Four men start at the same place to walk around a garden: A can go around in 9 minutes, B in 10 minutes, C in 12 minutes, and D in 15 minutes; in what time will they all meet at the starting-point? *Ans.* 180 minutes.

37. A, B, C, and D start from the same point, A travelling a mile in 18 minutes, B in 24 minutes, C in 30 minutes, and D in 35 minutes; what is the least whole number of miles each may travel that they may return to the starting-point at the same moment? *Ans.* A, 140; B, 105; C, 84; D, 72.

COMMON FRACTIONS.

1. Add $3\frac{1}{2}$ of $\frac{1}{8}$, $\frac{1}{14}$ of $3\frac{1}{2}$, and $\frac{1}{2}$ of $16\frac{2}{3}$ of $\frac{3}{80}$. *Ans.* 1.

2. Add $\frac{1}{10}$ of $\frac{3}{7}$ of $13\frac{2}{9}$, $\frac{1}{15}$ of $6\frac{3}{7}$ of $2\frac{2}{9}$, and $\frac{14}{15}$ of $3\frac{1}{7}$. *Ans.* $4\frac{19}{42}$.

3. Subtract $\frac{2}{3}$ of $\frac{3}{5}$ of $9\frac{1}{6}$ from $\frac{14}{45}$ of $\frac{6}{7}$ of 56. *Ans.* $11\frac{4}{15}$.

4. What is the value of $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$ minus $\frac{1}{4}+\frac{1}{6}+\frac{9}{10}$? *Ans.* $1\frac{17}{120}$.

5. Multiply $\frac{8}{9}$ of $\frac{1}{2}$ of $\frac{20}{36}$ by $24\frac{9}{15}$ of $\frac{75}{100}$. *Ans.* $4\frac{5}{9}$.

6. Multiply $\frac{21}{22}$ of $\frac{1}{20}$ of $14\frac{2}{3}$ by $\frac{5}{8}$ of $\frac{3}{10}$ of $\frac{1}{13}$ of $9\frac{3}{4}$ of 100. *Ans.* $9\frac{27}{32}$.

7. Divide $\frac{4}{5}$ of $\frac{7}{12}$ of $\frac{9}{10}$ by $\frac{5}{6}$ of $\frac{15}{64}$ of $\frac{21}{20}$ of $5\frac{1}{3}$. *Ans.* $\frac{48}{125}$.

8. Divide $\frac{7}{10}+\frac{17}{20}-\frac{19}{30}$ by $\frac{2}{3}+\frac{4}{5}-\frac{5}{6}$. *Ans.* $1\frac{17}{38}$.

9. Divide $\frac{15}{16}\times\frac{17}{21}\times\frac{19}{36}$ by $\frac{38}{63}\times\frac{17}{24}\times\frac{45}{63}$. *Ans.* $1\frac{5}{16}$.

10. What part of $3\frac{3}{4}$ is $2\frac{2}{5}$? Of $7\frac{3}{5}$ is $5\frac{1}{15}$? *Ans.* $\frac{16}{25}$; $\frac{2}{3}$.

11. Divide the sum of $4\frac{7}{8}$ and $5\frac{2}{3}$ by their difference. *Ans.* $13\frac{6}{19}$.

What is the value

12. Of $\frac{24}{25}\times\frac{75}{81}\times\frac{9}{64}\times\frac{256}{289}$? *Ans.* $\frac{32}{289}$.

13. Of $\frac{49}{64}\times\frac{27}{128}\times\frac{144}{729}\times\frac{48}{343}$? *Ans.* $\frac{1}{224}$.

14. Of $\frac{729}{961}\times\frac{992}{1089}\times\frac{231}{1024}\times\frac{124}{693}$? *Ans.* $\frac{27}{968}$.

15. Of $\frac{289}{441}\times\frac{231}{340}\times\frac{700}{841}\times\frac{870}{999}\times\frac{1998}{935}$? *Ans.* $\frac{20}{29}$.

16. Of $(\frac{1}{3}+\frac{8}{15})\times\frac{3}{11}+(\frac{4}{17}+\frac{5}{6})\times 3$? *Ans.* $3\frac{827}{1870}$.

17. Of $(\frac{10}{11}+\frac{15}{19})\times(\frac{21}{22}-\frac{35}{48})\times\frac{627}{710}\times\frac{44}{49}$? *Ans.* $\frac{17}{56}$.

18. Of $\frac{25}{57}\times\frac{42}{65}\times\frac{19}{20}\div\frac{7}{13}$? *Ans.* $\frac{1}{2}$.

19. Of $7\frac{7}{10}\times 4\frac{6}{11}+(4\frac{1}{2}-3\frac{4}{5})-(5\frac{6}{7}+6\frac{3}{11})$? *Ans.* $23\frac{439}{770}$.

20. Of $732\frac{1}{11}-(33\frac{1}{3}+37\frac{1}{2})-(6\frac{1}{4}+6\frac{7}{12})\times 49\frac{15}{16}$? *Ans.* $20\frac{415}{1056}$.

21. Of $(\frac{24}{5}-\frac{15}{11})\times\frac{77}{81}\div(6\frac{2}{3}-5\frac{3}{4})$? *Ans.* $3\frac{31}{55}$.

22. Of $(\frac{54}{63}+3\frac{4}{7})\times 9\frac{7}{9}\div(6\frac{2}{3}-4\frac{1}{12})$? *Ans.* $16\frac{16}{21}$.

23. Of $(77\frac{20}{21}-44\frac{3}{7})\times(6\frac{5}{8}-5\frac{3}{4})\div(8\frac{2}{3}+7\frac{3}{4}-3\frac{7}{12})$? *Ans.* $2\frac{2}{7}$.

24. Of $\dfrac{\frac{4}{7}\text{ of }\frac{3}{8}}{5\frac{1}{14}-4\frac{51}{56}}-\dfrac{\frac{5}{11}\text{ of }\frac{13}{15}}{7\frac{15}{22}-6\frac{1}{2}}$? *Ans.* 1.

25. Of $\dfrac{6\frac{1}{4}+5\frac{1}{2}}{6\frac{1}{4}-5\frac{1}{2}}+\dfrac{6\frac{1}{4}\times 5\frac{1}{2}}{6\frac{1}{4}\div 5\frac{1}{2}}$? *Ans.* $45\frac{11}{12}$.

26. Of $\dfrac{1+\frac{1}{5}}{1-\frac{1}{5}}\div\dfrac{1\times\frac{1}{5}}{1\div\frac{1}{5}}$? *Ans.* $37\frac{1}{2}$.

27. Of $\dfrac{1\frac{1}{2}\times 1\frac{1}{2}\times 1\frac{1}{2}\times 1\frac{1}{2}-1}{1\frac{1}{2}\times 1\frac{1}{2}\times 1\frac{1}{2}-1}$? *Ans.* $1\frac{27}{38}$.

28. Of $\left(\dfrac{\frac{1}{3}+\frac{3}{10}}{6\frac{1}{3}}+\dfrac{5\frac{5}{7}}{4\frac{4}{9}}\right)\div 11\times\dfrac{8\frac{1}{12}}{15\frac{2}{5}}$? *Ans.* $\frac{6}{25}$.

29. Of $\dfrac{1-\frac{1}{5}}{2}\times\dfrac{2-\frac{1}{5}}{3}\times\dfrac{3-\frac{1}{5}}{4}\div\dfrac{4-\frac{1}{5}}{5}$? *Ans.* $\frac{21}{95}$.

30. Of $\left(\dfrac{\frac{1}{3}+\frac{2}{5}}{3\frac{1}{5}}-\dfrac{8\frac{5}{8}}{115}\right)\div\left(3\times\dfrac{3}{81\frac{9}{37}}\right)$? *Ans.* $1\frac{47}{120}$.

31. What must I pay for $14\frac{12}{17}$ yards of cloth, if $47\frac{3}{11}$ yards cost \$$378\frac{6}{7}$? *Ans.* \$$117\frac{6}{7}$.

32. What must a farmer pay for $17\frac{3}{4}$ tons of hay, if $5\frac{5}{6}$ tons cost \$$70\frac{7}{9}$? *Ans.* \$$215\frac{11}{30}$.

33. What must I pay for $10\frac{4}{5}$ barrels of rice, if $7\frac{3}{5}$ barrels cost \$$72\frac{5}{6}$? *Ans.* \$$103\frac{1}{2}$.

34. If \$$24\frac{3}{4}$ will buy $25\frac{2}{3}$ bushels of corn, how many dollars will buy $50\frac{5}{9}$ bushels? *Ans.* \$$48\frac{3}{4}$.

35. If $26\frac{8}{11}$ bushels of wheat are given for $6\frac{8}{15}$ tons of hay, how many bushels must be given for $46\frac{1}{5}$ tons of hay? *Ans.* 189 bu.

36. If $7\frac{2}{9}$ yards of cloth are given for $12\frac{2}{15}$ bushels of corn, how much corn must be given for $19\frac{1}{21}$ yards of cloth? *Ans.* 32 bu.

37. If at a certain time of day a pole 63 feet long casts a shadow $36\frac{3}{4}$ feet long, what is the length of a pole which casts a shadow $33\frac{3}{5}$ feet long? *Ans.* $57\frac{3}{5}$ feet.

38. A cistern has a capacity of $289\frac{1}{11}$ gallons, and has a pipe discharging into it $25\frac{1}{3}$ gallons per hour, and there is a leak through which it loses $5\frac{1}{7}$ gallons per hour; how long will it take to fill the cistern? *Ans.* $14\frac{7}{22}$ hours.

39. Mr. Williams, laying in a stock of goods, invests $\frac{1}{3}$ of his money in flour, $\frac{1}{4}$ in sugar and molasses, $\frac{1}{12}$ in tea and coffee, and the remainder, \$540, in sundry other groceries; what was the whole amount invested? *Ans.* \$1620.

DECIMAL FRACTIONS.

1. What common fraction equals .00096? *Ans.* $\frac{3}{3125}$.

2. Express .06125 as a common fraction. *Ans.* $\frac{49}{800}$.

3. Add $\frac{.2}{2.7}$, $11\frac{\frac{1}{2}}{5.4}$, $2\frac{7}{45}$, and .1. *Ans.* $13\frac{19}{45}$.

4. Add $\frac{.4}{5}$, $\frac{2\frac{5}{6}}{5\frac{2}{3}}$, $\frac{21}{48}$, and $\frac{2\frac{4}{9}}{1\frac{7}{18}}$. *Ans.* $2\frac{311}{400}$.

Find the value of

	Ans.		*Ans.*
5. $.0075\times4.008$.	.03006.	**11.** $3\frac{6\frac{2}{3}}{11\frac{3}{7}}+\frac{5\frac{5}{9}}{28\frac{4}{7}}-.83\frac{1}{3}$.	$2\frac{17}{18}$.
6. $13\frac{22}{35}\div(28+\frac{1}{5}\div.7)$.	$\frac{53}{110}$.		
7. $118\frac{1}{4}\div.04\frac{1}{2}$.	$2627\frac{7}{9}$.	**12.** $\frac{3.1515+3.08\frac{3}{5}}{12\frac{12}{25}-.005}+\frac{3.5}{.35}$.	$10\frac{1}{2}$.
8. $14\frac{\frac{2}{3}}{27}\div11\frac{1.75}{13.5}$.	$1\frac{469}{1803}$.	**13.** $\frac{5}{7}$ of $.3\div.25$ of $\frac{8}{21}$.	$\frac{13}{42}$.
9. $\frac{2\frac{1}{4}}{3}+\frac{3}{16}-\frac{1\frac{1}{2}\times2\frac{1}{3}}{\frac{7}{4}\text{ of }\frac{16}{5}}$.	$\frac{5}{16}$.	**14.** $2.400\frac{1}{5}\div1.56\frac{1.25}{6\frac{1}{4}}$.	$1.53\frac{47}{71}$.
10. $\frac{7.5}{37\frac{1}{2}}$ of $\frac{2.5}{2\frac{2}{5}}+2\frac{19}{36}$.	$2\frac{53}{72}$.	**15.** $\frac{2\frac{1}{6}+4\frac{5}{12}}{1\frac{3}{4}\times3\frac{1}{3}}\div\frac{1}{6}$ of $.6\frac{5}{7}$.	10.

16. Multiply 1.25 of $.8\frac{3}{4}$ of $9\frac{6}{35}$ by .8 of $\frac{\frac{7}{8}}{.2}$. *Ans.* $35\frac{7}{64}$.

17. Divide £240.75 by $\frac{3}{4}$ of $\frac{24.5}{28}$ of $\frac{5}{7}$ of 1.8. *Ans.* £$285\frac{1}{3}$.

18. What number multiplied by 7.6 will give $6\frac{2}{3}$ for a product? *Ans.* $\frac{50}{57}$.

19. Divide seven millionths by twelve and a half ten-millionths. *Ans.* 5.6.

20. Add $\frac{1}{4}$, $\frac{8\frac{1}{2}}{36}$, .375, and .5, and multiply the result by $6.2\frac{1}{2}$. *Ans.* $8\frac{73}{144}$.

21. Subtract $\frac{3}{7}$ of 12 from $\frac{5}{8}$ of $\frac{8.75}{.4}$, and divide the remainder by $\frac{5}{7}$. *Ans.* $11\frac{301}{320}$.

22. Multiply the sum of .5 and $\frac{3.5}{14}$ by the difference between $\frac{8\frac{1}{2}}{17}$ and $\frac{10}{26\frac{2}{3}}$. *Ans.* $\frac{3}{32}$.

23. Multiply $\frac{62\frac{1}{2}}{1000}$ by 25 millionths, and divide the product by 125 hundred-thousandths. *Ans.* .00125.

Find the value

24. Of $.9\times\frac{18}{35}\times\frac{7}{24}\times\frac{16}{27}$. *Ans.* .08. **25.** Of $\frac{229}{1728}\times1.44$. *Ans.* $.1908\frac{1}{3}$.

26. Of $(6.05+3\frac{3}{4}-.004\frac{4}{5})\div.4$. *Ans.* 24.488.

27. Of $28\frac{.25}{.7}\div1134\frac{\frac{2}{15}}{\frac{3.5}{7.5}}$. *Ans.* $.02\frac{1}{2}$.

28. Of $\left(\frac{8-.4}{2}+\frac{16-.8}{4}\right)\times7\frac{7}{10}$. *Ans.* 58.52.

29. Of $.006006\div.024\frac{19.8}{24.2}$. *Ans.* .242.

30. Of $(.2\times.02)-(.01\times.001)\times10\frac{2}{5}$. *Ans.* .003896.

31. Of $13\frac{13}{189}\div\left(50-\frac{100}{3}+\frac{7\frac{1}{3}}{4.5}\right)$. *Ans.* $\frac{5}{7}$.

32. Of $(6.125+8.75-9.1235)\div.0125$. *Ans.* 460.12.

33. Of $\frac{7-3.004}{.2+7.3}\times\frac{5-.08}{4.8}\div1\frac{1}{5}$. *Ans.* .4551.

34. Of $(.101+.5\frac{1}{4}-.3\frac{1}{8})\times\frac{.04\frac{1}{6}}{.1\frac{2}{3}}\div.8\frac{4.2}{.6}$. *Ans.* .05225.

BILLS AND ACCOUNTS.

Let pupils make out bills and accounts from the following statements:

1. David Rowe & Co. bought of Nathaniel Rudolph 10 bbl. Yellow Sugar, 3031 lb., @ 4¢; 2 tierces Lard, 713 lb., @ 9½¢; 2 tierces Rice, 1216 lb., @ 7¢; what was the amount of the bill? *Ans.* $274.09½.

2. Required the footing of the following bill: 2 doz. Smith's Bitters, @ $7.25; 2 doz. Laudanum, 2 oz., @ $2.25; 2 doz. Brown's Syrup, @ $1.75; 2 doz. Fancy Soap, @ 67¢; 2 doz. Golden Liniment, @ $1.85. *Ans.* $27.54.

3. Mrs. Brown bought of Tyndale & Co., Jan. 1, 1895, the following articles: 1 Soup Tureen, $2.50; 2 Sauce Tureens, @ 75¢; 1 doz. Butter Plates, 50¢; 2 Glass Pitchers, @ 62½¢; 3 Oval Glass Dishes, @ 50¢; 4 Cov'd Dishes, @ 1.25; required the bill receipted. *Ans.* $12.25.

4. Mr. James Thornton of Peoria, Ill., bought of Hood, Bonbright & Co. the following articles, Oct. 14, 1894: 3 doz. Ladies' Berlin Gloves, @ $2.25; 1 doz. do. Berlin ½ Gauntlets, $3.75; 1 doz. Colored Buck do., $15; 1 doz. Ladies' Black Jouvin Kid Gloves, $16; 2 doz. Gents' Buck Driving Gauntlets, @ $16.50; 2 doz. Gents' White Kid Gloves, @ $11; make out bill for amount, deducting 20%. *Ans.* $77.20.

5. Mrs. Jane Wilson of Trenton, N. J., presented the following bill to Thomas Horne, March 1, 1895: Board for 4 weeks, @ $9; fuel and light, 4 weeks, @ $1.50; washing, 5¼ doz., @ $1. Mr. Horne presented the following bill to Mrs. Wilson at the same date: Feb. 15, 15 lb. Tea, @ 45¢; Feb. 19, 10 lb. Coffee, @ 25¢; 50 lb. Granulated Sugar, @ 5¢; Feb. 26, 28 lb. Butter (½ firkin), @ 35¢, and 4½ doz. Eggs, @ 42¢. Make out both bills, receipting the smaller and crediting its amount upon the other. *Ans.* Bal. $24.06.

DENOMINATE NUMBERS.

1. How many days from Jan. 1, 1880, to Jan. 1, 1900? *Ans.* 7305.

2. If a boy averages 32 inches each step, how many steps will he take in walking 1 mile? *Ans.* 1980 steps.

3. How many acr [illegible] square can be enclosed by 160 rods of fence? *Ans.* 10 acres.

4. A lot of silver weighs 432 lb. Avoirdupois; how much does it weigh by Troy weight? *Ans.* 525 lb.

5. April 10th falls on Wednesday; on what day of the week does the following 10th of September fall? *Ans.* Tuesday.

6. How many powders, each containing 5 grs. of quinine, can be made from 1 lb. Apothecaries'? From 1 lb. Av.? *Ans.* 1152; 1400.

7. On May 20th the sun rises at Philadelphia at 4 h. 40 min. A. M., and sets at 7 h. 13 min. P. M.; how long is it between sunrise and sunset? *Ans.* 14 h. 33 min.

8. A traveller finds his watch one hour slow; in which direction has he travelled, and how far directly east or west? *Ans.* 15° east.

9. At 56 lb. per bushel what will be the weight of 84 bu. 3 pk. 6 qt. of corn? *Ans.* $4756\frac{1}{2}$ lb.

10. If the distance between two places on the same parallel is 60°, and it is 12 M. at the eastern point, what is the time at the western point? *Ans.* 8 A. M.

11. If a bushel of wheat weighs 60 lb., how many sacks holding 2 bu. each will be required to hold 9 T. 7 cwt. 20 lb.? *Ans.* 156.

12. How far apart are two places whose difference of time is 2 h. 30 min., if 1° equals 60 miles? *Ans.* 2250 miles.

13. A farmer sowed 8 bu. 2 pk. 2 qt. of wheat, and harvested from it 443 bu. 3 pk. 4 qt. of grain; how much was raised from 1 bushel? *Ans.* $51\frac{115}{137}$ bu.

14. A teacher started for Europe on July 4, 1892, and returned on the following 10th of May, 1893; how many days was she away? *Ans.* 310.

15. The area of a rectangular field is 121 A. 100 P., and one side is 40.50 chains; what is the length of the other side of the field? *Ans.* $30\frac{5}{162}$ ch.

16. What is the difference between the standard and local time of Boston, long. 71° 3′ 30″? *Ans.* 15 min. 46 sec.

17. A car contains 21,643 lb. of wheat; find the value of the load at 92¢ per bushel, 1 bushel = 60 lb.? *Ans.* \$331.86 –.

18. When it is 9 o'clock P. M. Friday, standard time, in San Francisco, 122° 4′ 15″ W., what is the time in the city of London? *Ans.* 5 A. M. Saturday.

19. Franklin Park, Boston, contains 525 acres of land; what is it worth at $0.37½ a square foot? *Ans.* $8575875.

20. When it is noon at Paris it is 6 h. 49 min. 45 sec. A. M. at Philadelphia; what time will it be at Paris when it is noon at Philadelphia? *Ans.* 5 h. 10 min. 15 sec. P. M.

21. At the rate of 10½ miles an hour, how far will a ship sail from 4 h. 20 min. A. M. to 7 h. 45 min. P. M.? *Ans.* 161⅞ mi.

22. How many silver coins, each weighing 412½ gr., can be coined from a bar of silver weighing 16 lb. 8 oz. Avoir.? *Ans.* 280.

23. What is the value of a pile of wood 19.5 feet long, 8.4 feet wide, and 6.3 feet high, at $4.25 per cord? *Ans.* $34.26.

24. Two men were born on the first day of January, one in the year 1756, the other in 1822; both died 37 years after they were born, upon the first of January; required the difference of their ages. *Ans.* 1 day.

25. A train whose rate was 35 miles an hour went a certain distance in 12 h. 40 min.; how long would it take a train whose rate was 40 miles to go the same distance? *Ans.* 11 h. 5 min.

26. A grocer has a false balance by which he gives but 15 oz. to the pound; what is the real value of a barrel of sugar (250 lb.) for which he receives $28? *Ans.* $26.25.

27. A man was born the 29th of February, 1824, and died the 3d of March, 1860; how many birthdays did he see, and what was his exact age? *Ans.* 10; age, 36 yr. 4 da.

28. A coal-dealer bought 350 tons of coal, weighing 2240 lb. each, at $3.50 a ton, and sold it at $4.25 a ton, each ton weighing 2000 lb.; what was his profit? *Ans.* $441.

29. If a telegram is sent from Philadelphia, 75° 10′ W., to San Francisco, 122° 24′ 15″ W., at 9 h. 30 min. P. M., at what hour, local time, will it be received at San Francisco; at what hour standard time? *Ans.* 6 h. 21 min. 3 sec. P. M.; 6 h. 30 min. 40 sec.

30. A goldsmith bought an ingot of gold at $192 per lb., and sold it at $16 an ounce, using Avoirdupois weight both times; if the true weight of the ingot was 8 lb. Troy, how much did he gain by the fraud, gold being worth $16 an ounce? *Ans.* $421

31. Two men start from different places on the equator, and travel toward each other till they meet; on comparing their watches with the time of the place of meeting, it is found that the first is 45 minutes slow and the second 1 hr. 15 min. fast; how far apart were the points at which they started? *Ans.* 2074.8 mi.

32. A balloon started from Paris with despatches for Tours, and travelled 143 miles 256 rods, which it made at the rate of 51 miles 80 rods an hour; starting at 4 A. M., when did it alight? *Ans.* 6 h. 48 min. $21\frac{3}{41}$ sec. A. M.

33. The distance from a certain toll-gate east to a tavern is $3\frac{4}{11}$ miles; from the toll-gate west to a school-house is $45\frac{10}{33}$ rods; half way between the tavern and the school-house is a creek 100 yards wide; how far from the toll-gate to the middle of the creek; also to the farther bank of the creek?

Ans. 1 mi. 195 rd. $2\frac{11}{12}$ yd.; 1 mi. 204 rd. $3\frac{5}{12}$ yd.

PRACTICAL MEASUREMENTS.

1. How many acres are there in a square lot each of whose sides is 20 chains? *Ans.* 40 A.

2. A man has a lot of land 32 rods long which contains 1 acre; required its width. *Ans.* 5 rods.

3. A rectangular field containing 8 acres is 32 rods wide; what is the length of the field? *Ans.* 40 rods.

4. A city lot containing 800 square yards has a front of 60 feet; what is its depth? *Ans.* 120 feet.

5. How many yards of fence will be needed to enclose a rectangular field 60 rods long, containing 20 acres? *Ans.* $1246\frac{2}{3}$ yd.

6. Find the area in acres of a right triangle, the length of the sides being 18 rods, 24 rods, and 30 rods. *Ans.* 1 A. 56 P.

7. A turnpike 40 feet wide was run through a township; how many acres per mile did it occupy? *Ans.* $4\frac{28}{33}$ A.

8. A farmer has a 10-acre field in the form of a right triangle; if one side measures 200 yards, what is the length of the other? *Ans.* 484 yards.

9. How many cakes of ice, 4 ft. long by 2 ft. wide, can be cut from a rectangular pond 60 ft. long by 40 ft. wide? *Ans.* 300 cakes.

10. How many square inches in the entire surface of a cube whose dimensions are 15 inches? *Ans.* 1350 sq. in.

11. Find the entire surface of a block of marble 8 ft. long, 18 in. wide, and 4 in. thick. *Ans.* 30⅓ sq. ft.

12. How many flagstones, 5½ ft. long by 3 ft. wide, will be needed to lay a sidewalk ½ a mile long and 6 ft. wide? *Ans.* 960.

13. A courtyard, 42 ft. by 32 ft., is to be paved with flagstones measuring 6 ft. by 4 ft.; what will be the cost at $3.75 a flagstone? *Ans.* $210.

14. A farm 1 mile square is divided into square fields, each containing 40 acres; how many miles of fence will it require to enclose all the fields? *Ans.* 10 miles.

15. A man has a lot 320 ft. long and 210 ft. wide; how many square yards in a sidewalk 6 ft. wide around this lot just outside of it? *Ans.* 722⅔ sq. yd.

16. Find the area of a gravel-walk 6 ft. wide just inside a fence enclosing a field 320 ft. long and 210 ft. wide. *Ans* 690⅔ sq. yd.

17. A farmer wishes to enclose a lot 350 ft. long by 280 ft. wide with a tight board fence 6 feet high; how many square feet of boards will it require? *Ans.* 7560 sq. ft.

18. How many sheets of tin 6 in. by 4 in. will be required to cover a roof 60 ft. by 36 ft., no allowance being made for overlapping? *Ans.* 12960 sheets.

19. Mary's hoop was 3 ft. in diameter; how many yards would it travel in making 100 revolutions? *Ans.* 314.16 yd.

20. There is a circular park 100 rods in diameter, and within it is a circular lake 10 rods in diameter; what is the area of the park exclusive of the lake? *Ans.* 48 A. 95.46 P.

21. A cubic foot of water weighs 1000 oz.; what will a gallon (231 cu. in. = 1 gal.) of water weigh? *Ans.* 133$\frac{49}{72}$ oz.

22. Find the weight of a quart of mercury, considering mercury 13 times as heavy as water. *Ans.* 434$\frac{133}{288}$ oz.

23. A tank 8 ft. long and 5 ft. wide requires 21$\frac{7}{9}$ sq. yd. of lead to line its sides and bottom; how deep is it? *Ans.* 6 ft.

24. Find the weight of a plank 22 ft. 6 in. long, 1
and 2½ in. thick, at 40½ lb. to the cubic foot. *Ans.*

25. How many bushels (2150.42 cu. in. = 1 bu.) will a bin hold, the dimensions of which are 10 ft. long by $7\frac{1}{2}$ ft. wide by 5 ft. deep? *Ans.* 301.33 bu.

26. Mr. Wilson has in his house a cistern 8 ft. 4 in. long, 6 ft. 9 in. wide, and 4 ft. 6 in. deep; what is its capacity in gallons? *Ans.* $1893\frac{39}{77}$ gal.

27. How many Belgian blocks, averaging 6 in. × 12 in. on the surface, will be required to lay a pavement on the roadway of a street 500 yards long and 15 yards wide? *Ans.* 135,000 blocks.

28. How many bricks, 8 in. × 4 in., will be required to lay a pavement on a sidewalk 7 feet wide, extending along 4 lots, each having 18 ft. 6 in. front? *Ans.* 2331 bricks.

29. A has a circular garden and B a square one; the distance around each is 64 rods; which contains the most land, and how much? *Ans.* A's, 69.939 P.

30. Mr. Glass has a cylindrical cistern 8 ft. 6 in. in diameter and 5 ft. 9 in. deep; how many hogsheads (63 gal.) of water does it contain? *Ans.* 38.74+ hhd.

31. The house to which the above cistern belongs is 40 ft. long and 34 ft. wide, the eaves projecting 6 in. on each side, and all the water falling on the roof is conducted to the cistern; to what depth would it be filled by a shower in which $\frac{7}{8}$ of an inch of rain falls? *Ans.* 1 ft. $10\frac{1}{8}$+ in.

32. I have a yard 100 ft. square which I wish to pave, making a flagged walk 6 ft. wide around the outside, at 75¢ a square yard, and paving the rest with bricks, at \$9 ℔ M., allowing 4 bricks to the square foot; what will be the whole cost? *Ans.* \$466.78.

33. I have a bin 8 ft. 3 in. long, 3 ft. 6 in. wide, and 4 ft. 9 in. deep, filled with corn in the ear; how many bushels are there (Art. 340), and how many bushels when shelled, if 2 bushels on the ear make 1 bushel when shelled? *Ans.* $87\frac{39}{50}$ bu.; $43\frac{89}{100}$ bu.

34. A room contained 3 windows, which were curtained with brocatelle $\frac{1}{2}$ of a yard wide; 10 yards were required for each window, @ \$1.50, and the curtains were lined with silk $\frac{5}{8}$ of a yard wide, @ \$.87$\frac{1}{2}$; how many yards of silk were required, and what was the whole cost of the curtains? *Ans.* 24 yd.; \$66.

PERCENTAGE AND ITS APPLICATIONS.

1. A base-ball club won 18 games and lost 12 games; what per cent. of its games did it win? *Ans.* 60%.

2. A lady lost $750, which was 25% of what she then had; how much had she at first? *Ans.* $3750.

3. I sold a horse so that $\frac{4}{5}$ of the gain equalled $\frac{2}{15}$ of the cost; what was the gain per cent.? *Ans.* $16\frac{2}{3}$%.

4. A wheelman sold his old bicycle for $80, and lost $16\frac{2}{3}$%; what did it cost him? *Ans.* $96.

5. I bought goods amounting to $500 for $5\frac{1}{2}$% off for cash; what did I pay for them? *Ans.* $472.50.

6. On July 11, 1864, a U. S. greenback was worth but 38 cents in gold; what was the premium on gold? *Ans.* $163\frac{3}{19}$%.

7. A sold 20% of his land to B, and 25% of the remainder to C, and then had 120 acres; how much had he at first? *Ans.* 200 A.

8. I paid $225 for goods on which a discount of 10% was allowed; what was the gross price? *Ans.* $250.

9. What is the list price of an article for which I paid $60, after a discount of 20% was deducted? *Ans.* $75.

10. A piano marked $850 was sold for 25 and 10% off for cash; what was the selling price? *Ans.* $573.75.

11. What per cent. of the list price is paid by a buyer who receives a discount of 20 and 10 per cent.? *Ans.* 72%.

12. What is the list price of an article for which I pay $15.30, after a discount of 15 and 10% is deducted? *Ans.* $20.

13. What per cent. of the list price is paid by a buyer who receives a discount of 20, 10, and 5%? *Ans.* 68.4%.

14. A merchant marks an article $1.50, and sells it at a discount of 20%; what is his gain per cent. if the article cost him 95 cents? *Ans.* $26\frac{6}{19}$%.

15. An organ costing $250 was sold at an advance of 20%; if the marked price was $400, what deduction per cent. was made? *Ans.* 25%.

16. What price must be marked on cloth costing $2.50 a yard, so that a profit of 20% may be made when the cloth is sold at 20% less than the marked price? *Ans.* $3.75.

17. A man sold a piano for $450, which was 10% less than he asked for it, and his asking price was 20% more than the piano cost him; what was its cost? *Ans.* $416⅔.

18. A bookseller wishes to mark up his dictionaries, which he is now selling at $8, so that he can deduct 20% and yet receive the present price; what must be the marked price? *Ans.* $10.

19. The sum of A's and B's money is $2800, and 25% of A's equals 33⅓% of B's; how much has each? *Ans.* A, $1600; B, $1200.

20. A bought stock at 25% below par, and sold it at 20% above par; how much did he invest if he gained $1920? *Ans.* $3200.

21. Mr. Thompson asked for flour 20% more than cost, but sold it for 90% of the price asked; what did he gain per cent.? *Ans.* 8%.

22. What must I ask for goods which cost 40 cents a yard that, after falling 33⅓%, I may gain 10% on the value? *Ans.* 66¢.

23. If I retail flour at a gain of 25%, and sell at wholesale for 4% less than at retail, what is my gain per cent. at wholesale? *Ans.* 20%.

24. B's loss at wholesale was 5%, and his retail price was 20% more than his wholesale price; what was his gain at retail? *Ans.* 14%.

25. C's retail gain is 12½%, and his retail price is 5% more than his wholesale price; what is his gain at wholesale? *Ans.* $7\frac{1}{7}\%$.

26. A farmer bought 20 turkeys for $12, and lost 20% of them; how must the remainder be sold to gain 12½% by the transaction? *Ans.* 84⅜¢ apiece.

27. D lost 20% of a cargo of flour, and sold the remainder for a gain of 40%; did he gain or lose on the investment, and how much per cent.? *Ans.* Gained 12%.

28. A barrel of vinegar leaked away 25%; what per cent. must I gain on the remainder that I may gain 10 per cent. on the cost of the vinegar? *Ans.* 46⅔%.

29. A sold two dwellings for $3600 each: on one he gained 20%, and on the other he lost 20%; what was his gain or loss by the transaction? *Ans.* Loss, $300

30. A's money is 25% more than B's money; then B's is how many per cent. less than A's?

31. A lady sold her piano for $360, and cleared ⅙ of this money; what per cent. would she have gained by selling it for 5% more than she received? *Ans.* 26%.

32. What is my annual rate of profit on a ten-acre wood-lot which cost $100, the yearly growth of wood averaging 1 cord to the acre, and the market price $4.25 a cord, the cutting and hauling costing 75¢ a cord? *Ans.* 35%.

33. Miss Willard sold her house and lot for $5000, receiving $\frac{12}{13}$ as much for the house as for the lot; on the lot she gained 4%, and on the house she lost 4%; what was the gain or loss? *Ans.* No gain or loss.

34. My agent bought some horses for me: he paid $250 for keeping, his commission was $375, and entire bill $15625; what was the rate? *Ans.* 2½%.

35. I sold goods on commission at 6% through a broker, who charged me 2%; my commission, after paying the brokerage, was $468; required the sum paid to my employers. *Ans.* $10998.

36. An agent buys goods on commission at 2½%, charging 1½% for the money; the agent charges his employer $224; what is the amount of goods bought? *Ans.* $5600.

37. An agent receives 4% commission and 2½% for guaranteeing payment; he remits to his principal $7480; what does the agent receive? *Ans.* $520.

38. I send my agent at Havana $2562.50 to purchase cigars; how much will he invest, deducting his commission of 2½% on his purchase? *Ans.* $2500.

39. A merchant instructs his agent in Chicago to buy pork to the amount of $3500; how much must he remit to settle the bill, the charges being $15 and the agent's commission 1½%? *Ans.* $3567.50.

40. A real-estate agent, having sold a house, paid to the owner $4387.50 as the proceeds of the sale, after deducting 2½% as his commission; for what was the property sold? *Ans.* $4500.

41. If an agent's commission is $143.20 when he sells $5728 worth of goods, how much would it be when he sells $12,575 worth? *Ans.* $314.37½.

42. A creditor receives on a debt of $5720 a dividend of 60%, of which he pays his attorney 5%; he afterward receives 20%

more, of which he pays his attorney 6%; what is the net amount he receives? *Ans.* $4335.76.

43. A gentleman left an estate to be divided equally among 5 heirs, subject to an inheritance tax of 5%, which caused a deduction of $2560 from the whole amount; what did each receive? *Ans.* $9728.

44. An agent's commission for a certain time was $364; if his sales had amounted to $1500 more, his commission would have been $401.50; what was the amount of his sales? *Ans.* $14,560.

45. A landlord allows his agent 5% on his gross rentals, and receives a net rental of $9560.80; what is the value of his property if the gross rental is 8% of the value? *Ans.* $125,800.

46. A landlord received $992.60 as the net rental of a house after the agent had paid $147.40 for repairs and charged 5% commission on the gross rental; what was the gross rental? *Ans.* $1200.

47. A man owns 80 shares of stock ($50): the company declares a 5% dividend, payable in stock; how many shares will he then own? *Ans.* 84 shares.

48. Mrs. Clark received $300, payable in stock, as her share of a 4 per cent. dividend; how many shares, at $50 each, did she then own? *Ans.* 156 shares.

49. Mr. Wallace received 10 shares and $45 in money as his share of a 5% dividend; how many shares, at $50 each, did he then own? *Ans.* 228 shares.

50. I exchanged 56 shares of bank stock ($50), at 2½% discount, for 40 shares of gas stock ($100), at 1½% premium, paying the balance in cash; how much cash did I pay? *Ans.* $1330.

51. Harold bought 30 shares of stock, $50 each, at 97%; he sold ⅓ of it at 99½%, and the remainder at 101½%; what did he gain by the transaction? *Ans.* $57.50.

52. A broker exchanged 700 shares of stock ($100), at 5% discount, for United States bonds ($100), at 5% premium, paying $70 in money; how many did he get? *Ans.* 634 bonds.

53. Mr. Fish bought a number of shares of bank stock ($50), the discount at 5% being $200; ¼ of it he sold at par and the rest at 7% advance; what was the average gain on each share? *Ans.* $5.12½.

54. Mr. Westlake bought Pennsylvania R. R. stock ($50) at 49⅞ and sold it at 53¼; after paying brokerage he found he had a profit of $237.50; how many shares did he buy? *Ans.* 76.

55. Mrs. Warner has $10,000 Reading Railroad 6's, quoted at 103¾; would she increase or diminish her annual income if she should sell them and buy with the proceeds 7% State bonds at 110⅛? *Ans.* Increase, $58.

56. I bought a lot 50 ft. front and 85 ft. deep, at a ground-rent of $5.40 per ft. front; what would be the cost of the property, the ground-rent being 6% of it? *Ans.* $4500.

57. Wishing to meet a note for $5000, I directed my broker to sell sufficient West Phila. Pass. Railway stock ($50) to cover the note and brokerage; if the stock was selling at 78⅝, how many shares must he sell, and what is the surplus? *Ans.* 64 sh.; $24.

58. I sold 25 shares of Philadelphia National Bank ($100) at 156½, and directed my broker to invest the proceeds in Norristown R. R. stock ($50) at 99; what is the amount of investment, after deducting brokerage? *Ans.* 39 shares; $40.37½ surplus.

59. How many shares of North Penn. R. R. ($50), at 49, must be sold, that the proceeds, invested in Pennsylvania State 6's, at 115½, may give an income of $600, ¼% brokerage being charged on sale and purchase? *Ans.* 237 shares; $8.37½ surplus.

60. Mr. Jackson sold $15,000 Union Pacific 7's at 101⅛, and invested part of the proceeds in Illinois 6's at 117½, sufficient to produce an income of $750, and deposited the remainder (brokerage ¼%) in bank; what was his bank deposit? *Ans.* $412.50.

61. I had some California 7's, which brought me in an income of $546, but, preferring an investment nearer home, I decided to exchange them for Philadelphia 6's; if the California bonds were worth 117 and the Philadelphia 105, how much must I add to my investment to secure the same income, brokerage not considered? *Ans.* $429.

62. I made $5000 by a speculation, and, wishing to invest it permanently, I bought $2000 6% bonds at 117⅜, and invested all the remainder possible in 4½% bonds at 110¼ (denominations as low as $50); what surplus remained, brokerage ¼%, and what was the annual income? *Ans.* Sur., $50.75; Inc., $225.75.

INTEREST, DISCOUNT, ETC.

1. The amount of a certain principal for a certain time at 4% is \$819, and at 8% for the same time is \$988; required the time and the principal. *Ans.* $6\frac{1}{2}$ yr.; \$650.

2. The amount of B's money for 8 years at 6% is \$5100 more than the interest of his money for 10 years at 8%; required B's money. *Ans.* \$7500.

3. The amount of \$250 for 6 years at 10% is to be divided between C and D, so that C shall have three times as much as D; what does each receive? *Ans.* C, \$300; D, \$100.

4. A, B, and C together have \$1200, of which A has twice, and B three times, as much as C; what is the interest of each for 5 years at 6%? *Ans.* A, \$120; B, \$180; C, \$60.

5. If the interest of \$2500 for 4 years, at 10 per cent., be divided into two parts, which are as 2 to 3, it will respectively give $\frac{1}{3}$ of B's and $\frac{1}{2}$ of A's money; how much has each? *Ans.* Each \$1200.

6. The interest on $\frac{1}{2}$ of A's and $\frac{1}{3}$ of B's fortune for 5 years, at 6 per cent., is \$240; what is the fortune of each, provided $\frac{1}{2}$ of A's equals $\frac{1}{3}$ of B's? *Ans.* A, \$800; B, \$1200.

7. A's money is four times B's, and the sum of the interest received by both for 3 years, at 8 per cent., is \$600; how much money has each? *Ans.* A, \$2000; B, \$500.

8. The interest for 4 years, at 5 per cent., on the money Martin owes is \$40, and the interest for the same time and rate per cent. on the money due him is \$70; how much more has he due than he owes? *Ans.* \$150.

9. The interest on the money A paid for a farm, house, and store for 8 years, at 5 per cent., equals \$18,000; what was the cost of each, provided the farm cost three times as much as the house, and the house twice as much as the store? *Ans.* Store, \$5000.

10. A man wishes to place such a sum of money on interest at 6 per cent. that it will give an annual interest of \$360 for a poor sister; required the amount invested. *Ans.* \$6000.

11. Two-thirds of A's fortune, plus $\frac{3}{4}$ of B's, being on interest for 6 years, at 5 per cent., amounts to \$7800; what is the fortune of each, if $\frac{2}{3}$ of A's equals $\frac{3}{4}$ of B's? *Ans.* \$4500; \$4000.

12. A gives his note for $850, payable in 2 yr. 8 mo. without interest; at the end of 8 mo. he wishes to pay the note; what should the holder of the note receive, money worth 6%? *Ans.* $758.93 –.

13. A man owes $600, of which one-third is to be paid in one year and the remainder in two years; what is the present value of the note, money worth 6 per cent.? *Ans.* $545.82.

14. What is the present worth of $2400, one-fourth due in 8 mo., one-third in 1 year, and the remainder in 18 mo., money being worth 6 per cent.? *Ans.* $2249.07.

15. A man owes $1800, $\frac{1}{3}$ of which is due in 1 year, $\frac{1}{2}$ of the remainder in 2 years, and the remainder in 3 years; required the present value, money worth 6%. *Ans.* $1610.225.

16. Required the cost of a 3 mo. draft on Philadelphia, exchange at $\frac{3}{4}$% discount, which will pay a debt of $950, money worth 6%. *Ans.* $928.15.

17. A Baltimore merchant wishes to pay a debt of $1500 in Detroit by a sight draft on the First National Bank, Baltimore; if exchange on Baltimore is $\frac{1}{8}$% premium at Detroit, what must be the face of the draft? *Ans.* $1498.13 –.

18. If the Baltimore merchant in the previous problem buy, instead of a sight draft, a 90-day draft, what will the time draft cost? *Ans.* $1474.91.

19. A Boston merchant sends to a creditor in Savannah a sight draft on Boston for $1498.13; what was the debt, exchange on Boston being at a premium of $\frac{1}{8}$%? *Ans.* $1500.

20. My agent sold $6000 worth of goods on commission at 3%, and remits to me the proceeds in a draft bought at $\frac{1}{2}$% premium, which I sell at $\frac{3}{4}$% premium; what did the goods bring me in? *Ans.* $5834.47.

RATIO AND PROPORTION.

1. A piece of cloth measured with a yard-stick 1 in. too short appears to be 25 yd. long; what is its true length? *Ans.* $24\frac{11}{36}$ yd.

2. A train which runs 40 miles an hour leaves New York at 8.45 A. M.; how far will it [illegible] miles.

3. A can do a piece of work in 6 days and B in 8 days; if A's wages are $2.50 a day, how much should B receive a day? *Ans.* $1.87½.

4. If 27 men build 54 rods of wall in 6 days, how many rods will 32 men build in 9 days? *Ans.* 96 rods.

5. If 5 men can mow 45 acres of grass in 6 days, in how many days will 12 men mow 90 acres? *Ans.* 5 days.

6. How many hours must 9 men work so that they may do as much work in 16 days as 12 men can do in 15 days of 8 hours each? *Ans.* 10 hours.

7. In what time will 8 masons build a wall 84 feet long, working 10 hours a day, if 12 masons build a wall 96 feet long in 8 days, working 8 hours a day? *Ans.* $8\frac{2}{5}$ days.

8. 25 men can build the walls of a house in 24 days; after they have worked 8 days, how many men must join them to finish it in 10 days? *Ans.* 15 men.

9. If a garrison of 450 men has provisions for 10 months, how many men must leave at the end of 4 months, so that the provisions remaining may last 9 months longer? *Ans.* 150 men.

10. If 300 cats kill 300 rats in 300 minutes, how many cats will kill 100 rats in 100 minutes?

11. If it costs $1845 to pave a street 750 ft. long and 60 ft. wide, what will be the cost of paving a street 1250 ft. long and 80 ft. wide? *Ans.* $4100.

12. If 8 men can reap a field of 40 acres in 12 days, working 8 hours a day, how long will it take 10 men to reap a field of 50 acres, working 9 hours a day? *Ans.* $10\frac{2}{3}$ days.

13. If 11 men, working 9 hours a day, can reap a field of 30 acres in 6 days, in how many days can 9 men, working 10 hours a day, reap a field 550 yards long and 420 yards wide? *Ans.* $10\frac{1}{2}$ days.

14. If 12 men, working 10 hours a day, can hoe 50 acres in 25 days, how long will it take 30 boys, working 8 hours a day, to hoe 75 acres, 8 men being equal to 12 boys? *Ans.* $28\frac{1}{8}$ days.

15. In a cotton-factory it was found that 6 men do as much as 8 boys, and 6 boys do as much as 9 girls; how many girls will be required to do as much as 54 men? *Ans.* 108 girls.

16. A can do 2 times as much as B in a day, B can do 3 times as much as C, C can do 2 times as much as D, and D can do ⅓ as much as E; in what time can E do as much as A does in 24 days? *Ans.* 96 days.

17. How much will 150 horses cost, if 10 horses are worth 24 cows, 3 cows are worth 18 sheep, 16 sheep are worth 15 pigs, and 20 pigs are worth $100? *Ans.* $10125.

18. If John can read a book of 320 pages, each page containing 32 lines and each line averaging 14 words, in 8 hours, how long will it take him to read a book of 480 pages, each page containing 35 lines and each line 16 words? *Ans.* 15 hours.

19. A, B, and C enter into partnership; A puts in $680, B $720, C $600; they gain $600 in 6 months; what does each receive? *Ans.* A, $204; B, $216; C, $180.

20. A and B agree to do a piece of work for $310; A sends 20 men for 12 days, and B sends 25 boys for 15 days; what shall each receive, if 3 men do as much as 5 boys? *Ans.* A, $160; B, $150.

21. A, B, and C entered into partnership and gained $740: A had $1200 in trade 9 mo., B $1400 in trade 8 mo., and C $1500 in trade 10 mo.; what was the gain of each? A, $216; B, $224; C, $300.

22. A borrowed $1200 for 6 mo. and $1500 for 8 mo.: at the end of 4 months he paid $2000; when, in equity, should the remainder be paid? *Ans.* 1 yr. 4 mo.

23. A, B, and C gained in partnership $3600; A's stock was $5000, B's $6000, and C's gain $1400; required C's stock and A's and B's gain. *Ans.* C, $7000; A, $1000; B, $1200.

24. Two men, A and B, gain 12½% on their stock, and then 20% of A's gain equals $520, and ⅔ of B's stock equals ¾ of A's; what is the stock of each? *Ans.* A, $20,800; B, $23,400.

25. A, B, and C enter into partnership with a capital of $240,000: A's stock was $65,000, B's gain $3400, and C's gain $3600; required A's gain and B's and C's stock. *Ans.* A, $2600; B, $85,000; C, $90,000.

26. A's capital was in trade 10 mo., B's 15 mo., and C's 18 mo.: A's gain was $1250, B's gain was $1500, C's gain $1350, and the whole capital invested was $6000; what was the capital of each partner? *Ans.* A, $2500; B, $2000; C, $1500.

27. A, B, and C form a partnership for carrying on a nursery: A contributes $800, B $600, and C 10 acres of land on which to establish the nursery; their first year's profits are \$1500, of which C receives \$660; what are A's and B's gain, and the value of C's land per acre? *Ans.* A's, \$480; B's, \$360; \$110 per acre.

28. An eccentric old schoolmaster made a will which read as follows: "I bequeath \$4059 to my two sons in the proportion of $\frac{5}{6}$ to $\frac{7}{8}$, which amounts are respectively equal to $\frac{9}{10}$ and $\frac{11}{12}$ of the amounts I bequeath to my two daughters;" required the share of each. *Ans.* \$1980; \$2079; \$2200; \$2268.

INVOLUTION AND EVOLUTION.

1. The 4th power of a number divided by the square of the number equals 49; what is the number? *Ans.* 7.

2. The square of a number divided by $\frac{2}{3}$ of the number equals 27; what is the number? *Ans.* 18.

3. A number divided by 6 gives double the square root of the number; what is the number? *Ans.* 144.

4. The square of a number multiplied by one-half of the number equals 32; what is the number? *Ans.* 4.

5. How many rods long is a square field containing 90 acres? How many yards of fence will enclose it? *Ans.* 120 rd.; 2640 yd.

6. The surface of all the faces of a cube equals 54 sq. in.; what is the length of the side of the cube? *Ans.* 3 in.

7. How far apart are the opposite corners of a square farm that contains 250 acres? *Ans.* 282.84+ rods.

8. How much shorter distance does a man walk in going diagonally across a rectangular field, 90 feet by 120 feet, than by going from one corner around to the opposite corner? *Ans.* 60 ft.

9. What is the distance from the lower corner to the opposite upper corner of a room 20 feet long by 16 ft. wide by 10 ft. high? *Ans.* 27.49+ ft.

10. A house is 36 feet from the ground to the eaves; how long must a ladder be to reach to the eaves if its foot is placed 15 feet from the house? *Ans.* 39 ft.

11. A ladder 85 feet long stands close against a building; how far must it be drawn out at the base that the top may be lowered 10 ft.? *Ans.* 40 ft.

12. A tree was broken 45 feet from the top, and fell so that the end struck the ground 27 feet from the foot; required the original height of the tree. *Ans.* 81 feet.

13. A ladder 65 ft. long, standing with its foot in the street, will reach on one side to a window 52 ft. high, and on the other to a window 60 ft. high; what is the width of the street? *Ans.* 64 ft.

14. If a horse tied to a post in the centre of a field by a rope 39.215 yards long can graze over an acre, how long a rope will enable him to graze over $2\frac{7}{9}$ acres? *Ans.* 65.358⅓ yd.

15. If a pipe 2 in. in diameter pour 40 gal. into a cistern in a given time, how much will a pipe 2½ in. in diameter pour in in the same time? *Ans.* 62½ gal.

16. A has a circular garden whose diameter is 15 rods, and B has a circular garden whose area is $2\frac{7}{9}$ times as great; what is the diameter of B's garden? *Ans.* 25 rods.

17. Mary has a circular garden, and Sarah a square one, and each of them contains 4 acres; how much greater is the distance around one than around the other? *Ans.* 11.514 rods.

18. A and B start from one corner of a park a mile square: A goes around the sides, and B goes straight across to the opposite corner, where they meet; how much farther did A travel than B? *Ans.* 187.452 rods.

19. A general, wishing to draw up his division into a square, found on the first trial he lacked 144 men to complete the square; he then diminished the side of the square by 2 men, and had 204 men over; how many men were there in the division? *Ans.* 7600.

20. A gentleman has a box whose edges are in the proportion of 2, 3, and 4, and its contents are 3000 cubic inches; what are the dimensions of the box? *Ans.* 10, 15, 20.

21. If a ball 7 inches in diameter weighs 13⅓ lb., what is the diameter of a ball which weighs 106⅔ lb., provided the materials of both are the same? *Ans.* 14 in.

22. There are three balls whose diameters are respectively 3 in., 4 in., and 5 in.; required the diameter of a ball of the same material weighing as much as the three. *Ans.* 6 in.

MENSURATION.

1. How many square yards in the entire surface of a cube whose solid contents are 216 cubic yards? *Ans.* 216 sq. yd.

2. Required the length of a hand-rail for a flight of stairs of 18 steps, each step being 7 in. high and $9\frac{1}{3}$ in. wide. *Ans.* $17\frac{1}{2}$ ft.

3. The entire surface of a square prism is 1650 sq. in.: one side of the base is 15 in.; what is the lateral surface? *Ans.* 1200 sq. in.

4. The lateral surface of a square prism is 600 sq. ft., the altitude is 15 ft.; what are the contents? *Ans.* 1500 cu. ft.

5. What is the difference between the area of a circle 12 inches in diameter and that of the inscribed square? *Ans.* 41.097 +.

6. How many square inches in the surface of a frame 3 in. wide around a looking-glass 8 in. in diameter? *Ans.* 103.67 + sq. in.

7. A gentleman has a block in the form of a parallelopiped which is 48 in. long, 36 in. wide, and 24 in. high; what is the entire surface of the block? *Ans.* 52 sq. ft.

8. Two steamers sail from the same port: one sails north at the rate of 15 miles an hour, and the other east at the rate of 12 miles an hour; how far apart are they in a day? *Ans.* 461.016 mi.

9. What are the contents in liquid measure of a tin pan whose height is 5 inches, upper diameter 17 inches, and lower diameter 9 inches? *Ans.* $2\frac{74}{77}$ + gal.

10. What is the length of a tape that will wind spirally around a cylinder 52 feet long and 3 feet in circumference, provided it passes around the cylinder once every 4 feet? *Ans.* 65 feet.

11. A rectangular field containing $7\frac{1}{2}$ acres is 30 rods wide; how much will it cost to build a fence from one corner to the diagonally opposite corner at $2 a rod? *Ans.* $100.

12. Having a square yard which contains $\frac{2}{5}$ of an acre, I make a gravel-walk around it which occupies $\frac{15}{64}$ of the whole yard; what is the width of the walk? *Ans.* 8 ft. 3 in.

13. A circular flower-bed, 15 feet in diameter, is surrounded by a walk 5 feet wide; how many square feet of surface does the walk contain? *Ans.* 314.16 sq. ft.

14. A coal-dealer has a wagon which holds exactly one ton of Schuylkill gray-ash coal (36 cu. ft.); if the wagon-bed is $6\frac{1}{2}$ ft. and 4 ft. wide, what is its depth? *Ans.* $16\frac{8}{13}$ in.

15. How many square feet of boards will it take to cover the gable ends of a house 34 ft. wide, 45 ft. high to the ridge-pole, and 30 ft. high to the eaves? *Ans.* 510 sq. ft.

16. If a crib 20 ft. long, 6 ft. wide, and 8 ft. high is filled with corn in the ear, how many bushels will there be when it is shelled if 2 bu. in the ear produce 1 bu. shelled? *Ans.* 307$\frac{1}{5}$ bu.

NOTE.—In Ex. 16 notice that corn in the ear is measured by heaped bushels, Art. 340.

17. The outer edge of a walk surrounding a circular plot of ground measures 120 feet, the inner edge measures 81 feet; how many square feet in the walk? *Ans.* 623.83 – sq. ft.

18. What is the expense of sodding a plot of ground 45 yd. long and 95 ft. wide with sods 15 in. × 24 in., the sods when laid costing $1.50 per hundred? *Ans.* $76.95.

19. How much will it cost to fence a rectangular garden 20 rods long and 15 rods wide, with pickets 4 inches wide and 3 inches apart, at $9 ℔ M.? *Ans.* $17.82.

20. What will be the cost of flooring, at $33.25 per M., of a three-story house, the inside measure being 58 ft. × 34 ft., deducting 15 ft. 6 in. by 8 ft. 3 in. for the stairs? *Ans.* $183.95.

21. How much will it cost to roof a warehouse, 48 ft. × 60 ft., with slate, the height of the ridge being 10 ft. and the eaves projecting 6 inches, at $14.75 per square (100 sq. ft.)? *Ans.* $469.05.

22. A lady, having a lot of ground 40 rods square, planted 2 acres with corn, 150 square rods with vegetables, 12 rods square with flowers, and the remainder she kept to pasture a cow; how much of the lot was pasture? *Ans.* 6 A. 26 P.

23. The earth taken from 4 cellars was used in grading a lot of ground; if the cellars were 30 × 21 ft., 28 × 18 ft., 24 × 16 ft., and 32 × 24 feet respectively, and 5 ft. deep, how many loads (cu. yd.) did it take? *Ans.* 423$\frac{1}{3}$ loads.

24. A yard 36 feet square has in the centre a fountain, the basin of which is 12 feet in diameter: there is a flower-bed, 4 feet wide, around three sides of the yard; what will be the expense of paving the remainder at $2.25 per square yard? *Ans.* $195.73.

25. What will be the cost of a thousand tiles in the shape of a rhombus 15 in. on a side, a line drawn from an obtuse angle perpendicular to the opposite side meeting it 9 in. from the acute angle, at 75¢ a square foot? *Ans.* \$937.50.

26. In a circular grass-plot whose diameter is 50 yards there is a gravel-walk, 1 yard wide, running round it 1 yard within the edge; what will be the cost of sodding the lot at 12¢ per square yard? *Ans.* \$217.90.

27. A circular room, of which the diameter is 25 ft. and height of wall 14 ft., is covered with a hemispherical dome; what is the cost of plastering the wall and dome at 25¢ per sq. ft.? *Ans.* \$520.32¾.

28. A cast-iron garden-roller is 40 in. long, 22½ in. in diameter, and the iron is ¾ in. thick; required its weight, if a cubic inch of iron weighs 4½ ounces. *Ans.* 576.532$\frac{11}{16}$ lb.

29. If a cannon-ball, 6 in. in diameter, is melted and cast in a conical mould 6 in. in diameter at the base, what is the length of the cone? *Ans.* 12 in.

30. A railroad embankment has an average depth, for 5 miles, of 12 feet, width at top 8 yards, and at bottom 12 yards; what was the cost of carting at 15¢ per load (cu. yd.)? *Ans.* \$52800.

31. An ice-house is 40 ft. long, 30 ft. wide, and 20 ft. deep; what area of ice 6 in. thick will be required from a pond to fill it? what would it weigh at 56 lb. the cu. ft.? *Ans.* 48000 sq. ft.; 1344000 lb.

32. A company wish to excavate a canal 25 miles long, with an average width of 9 yards and an average depth of 5 ft. 6 in.; how long will it take 500 men to do the work, if each man averages 12 cubic yards a day? *Ans.* 121 days.

33. In a tin funnel, one part conical, the slant height of the conical part is 4 in., the circumference at one end 10 in., and at the other end 1 in.; the other part is cylindrical, the length being 5 in.; required the number of sq. in. of tin in it. *Ans.* 27 sq. in.

34. A has a circular field 72 rods in diameter enclosed with a wire fence, and B has a square field requiring the same length of fence to enclose it; what is the difference in the areas of the two fields? *Ans.* 873.75 P.

35. What costs the excavation for a cellar 5½ ft. deep under the main part of a dwelling-house, 32×24 ft., and an excavation, 1½ ft. wide and 2 ft. deep, for the walls of a wing 16 ft. square, at 50¢ per cu. yd.? *Ans.* \$80.72$\frac{2}{9}$.

SUPPLEMENT.

[The *Supplement* contains additional matter for advanced classes, or for schools requiring more than the standard course in arithmetic.]

615. The following methods of greatest common divisor and least common multiple are often used:

1. Find the greatest common divisor of 42, 84, and 126.

SOLUTION.—We write the numbers one beside another, as in the margin. Dividing by 2, we see that 2 is a factor of each number; it is therefore a factor of the G. C. D. (Prin. 1). Dividing the quotients by 3, we see that 3 is a factor of each number, and therefore a factor of the G. C. D.; and in the same way we see that 7 is a factor of the G. C. D. Now, since the quotients 1, 2, and 3 are prime to each other, 2, 3, and 7 are all the common factors; hence their product, which is 42, is the G. C. D.

OPERATION.

$$\begin{array}{r|l} 2 & 42 - 84 - 126 \\ \hline 3 & 21 - 42 - \ \ 63 \\ \hline 7 & \ \ 7 - 14 - \ \ 21 \\ \hline & \ \ 1 - \ \ 2 - \ \ \ \ 3 \end{array}$$

$$2 \times 3 \times 7 = 42$$

2. Find the least common multiple of 12, 30, and 70.

SOLUTION.—Placing the numbers one beside another and dividing by 2, we see that 2 is a factor of each of them; it is therefore a factor of the L. C. M. (Prin. 3); dividing the quotients that will contain it by 3, we see that 3 is a factor of some of the numbers; it is therefore a factor of the L. C. M. Dividing the next quotients by 2, we see that 5 is a factor of some of them; hence 5 is a factor of the L. C. M., and, the quotients having no other common factor, we see that the factors of the given numbers are 2, 3, 5, 2, and 7; hence their product, which is 420, is the L. C. M.

OPERATION.

$$\begin{array}{r|l} 2 & 12 - 30 - 70 \\ \hline 3 & \ \ 6 - 15 - 35 \\ \hline 5 & \ \ 2 - \ \ 5 - 35 \\ \hline & \ \ 2 - \ \ 1 - \ \ 7 \end{array}$$

$$2 \times 3 \times 5 \times 2 \times 7 = 420$$

SIMPLE INTEREST BY CANCELLATION.

616. The following Six Per Cent. Method by *Cancellation* will be found convenient and practical:

Rule for Months.—*Point off two places in the principal, divide by 2, and multiply by the number of months.*

For the Int. for 2 mo. is $\frac{1}{100}$ of the principal, and for 1 mo. it is ½ of $\frac{1}{100}$ of the principal; hence the above rule.

Rule for Days.—*Point off three places in the principal, divide by 6, and multiply by the number of days.*

For the Int. for 2 mo., or 60 days, is $\frac{1}{100}$ of the principal, and for 6 da. it is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of the principal, and for 1 da. it is $\frac{1}{6}$ of $\frac{1}{1000}$ of the principal; hence the above rule.

1. What is the Int. of \$2400 for 38 mo. at 7%?

SOLUTION.—We point off two places in \$2400 to divide by 100, then divide by 2, and multiply by 38, then divide by 6 to find the Int. at 1%, and multiply by 7 to find the Int. at 7%; reducing by cancellation, we have \$532.

OPERATION.

$$\frac{\$\overset{2}{\cancel{24}}.00\times 38\times 7}{\cancel{2}\times\cancel{6}}=\$532$$

2. What is the Int. of \$4800 for 63 da. at 6%? at $4\frac{1}{2}$%?

SOLUTION.—The Int. for 1 da. is $\frac{1}{6}$ of $\frac{1}{1000}$ of the Prin.; hence we point off three places in \$4800, divide by 6 to find the Int. for 1 da., and multiply by 63 to find the Int. for 63 days; cancelling and multiplying, we have \$50.40.

OPERATION.

$$\frac{\$\overset{.800}{\cancel{4}.\cancel{8}\cancel{0}\cancel{0}}\times 63}{\cancel{6}}=\$50.40$$

NOTE.—To find the Int. at $4\frac{1}{2}$%, divide by 6 and multiply by $4\frac{1}{2}$, expressing the work and cancelling.

ANNUAL INTEREST.

617. Annual Interest is the simple interest of the principal and of each year's interest from the time of its accruing until settlement.

618. Annual Interest is sanctioned by some States when the note is written "with interest payable annually."

1. *Annual Interest* allows interest on the *unpaid interest* of a debt as well as upon the debt itself.

2. The neglect to collect the annual interest on a note drawn "with interest payable annually" is, in some States, regarded as a waiving of the contract requiring it.

1. What is the amount due on a note of \$500, at 6%, for 3 yr. 3 mo., interest payable annually?

SOLUTION.—The interest on \$500 for 1 year is \$30, and for 3 yr. 3 mo. is \$97.50. The first year's interest is on interest 2 yr. 3 mo., the 2d year's interest, 1 yr. 3 mo., and the 3d year's interest only 3 mo., which is equivalent to 1 year's interest on interest for $2\frac{1}{4}$ yr. + $1\frac{1}{4}$ yr. + $\frac{1}{4}$ yr., or $3\frac{3}{4}$ yr. The interest of \$30 for $3\frac{3}{4}$ yr. equals \$6.75; adding the interest on the principal, the interest on the interest, and the principal, we have \$604.25, the amount due.

OPERATION.

$2\frac{1}{4}$ yr. + $1\frac{1}{4}$ yr. + $\frac{1}{4}$ yr. − $3\frac{3}{4}$ yr.

\$500 × .06 = \$30, int. for 1 yr.

\$30 × $3\frac{1}{4}$ = \$97.50, int. on prin. for $3\frac{1}{4}$ yr.

\$30 × $.22\frac{1}{2}$ = 6.75, int. on int. for $3\frac{3}{4}$ yr.

500 principal.

\$604.25, amount.

Rule.—I. *Find the interest on the principal for the given time and rate; also find the interest on each year's interest for the time it has remained unpaid.*

II. *The sum of these interests will be the annual interest, and this, added to the principal, will be the amount due.*

WRITTEN EXERCISES.

2. Find the annual int. of $2700 for 3 yr. at 6%. *Ans.* $515.16.

3. Find the annual int. of $5000 for 7 yr. at 4½%. *Ans.* $1787.625.

4. Find the annual int. of $780 for 3 yr. 5 mo. at 5%. *Ans.* $141.53¾.

5. What is the annual interest and the amount of $874.50 for 5 yr. 6 mo. 18 da. at 4%? *Ans.* $211.98–; $1086.48–.

6. What is the difference between the annual interest and the simple interest of $7500 for 5 yr. 8 mo. at 6%? *Ans.* $360.

7. What is the amount due Jan. 31, 1895, on a note for $2500, dated July 10, 1890, interest payable annually at 4%, if the yearly interest has been regularly paid? *Ans.* $2556.94⅓.

COMPOUND INTEREST.

619. Compound Interest is interest on both principal and interest when the interest is not paid when due.

1. Compound interest assumes that if the borrower does not pay the interest when due, it is proper that he should pay interest for it until paid. While it seems just, it has not the sanction of law.

2. Many savings-banks allow compound interest upon balances remaining on deposit for a full interest period.

1. What is the compound interest of $500 for 2 yr. at 4%?

SOLUTION.—Multiplying by the rate per cent., we find the interest for 1 year to be $20; adding this to the principal, we find the amount to be $520, which is the principal for the 21 year. Multiplying the new principal by the rate, we find the interest for the 2d year to be $20.80, and adding this to the 2d principal, we find the amount for 2 years to be $540.80, from which subtract the first principal, and the remainder, $40.80, is the compound interest. Hence the following

OPERATION.

$$
\begin{array}{rl}
\$500 & \\
\underline{.04} & \\
20.00 & = \text{Int. 1st year.} \\
\underline{500} & \\
520.00 & = \text{Am't 1st year.} \\
\underline{.04} & \\
20.80 & = \text{Int. 2d year.} \\
\underline{520} & \\
540.80 & = \text{Am't 2d year.} \\
\underline{500} & \\
40.80 & = \text{C. Int. for 2d yr.}
\end{array}
$$

Rule.—I. *Find the amount of the principal for the first period of time for which interest is reckoned, and make this the principal for the second period.*

II. *Find the amount of this principal for the next period, and thus continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the result will be the compound interest.*

WRITTEN EXERCISES.

2. Find the comp. int. of $375 for 3 yr. at 6%. *Ans.* $71.63.

3. Find the comp. int. of $800 for 5 yr. at 7%. *Ans.* $322.04.

4. Find the comp. int. of $643.80 for 3 yr. 6 mo. at 4%. *Ans.* $94.87.

5. What is the compound interest of $2050.50 for 2 yr. 6 mo. at 6%, payable semi-annually? *Ans.* $326.59.

6. What is the amount of $350 for 1 yr. 9 mo. at 8 per cent., payable quarterly? *Ans.* $402.04.

7. What is the amount of $600 for 10 yr. at 6%, compound interest?

SOLUTION.—In the table (page 415), under 6%, and opposite 10 yr., we find the amount of $1 to be $1.7908477; $1.7908477 × 600 = $1074.51.

8. What is the amount of $640 for 12 yr. at 7 per cent., compound interest? *Ans.* $1441.40.

9. What is the amount of $400 for 18 yr. at 4 per cent., compound interest? *Ans.* $810.33.

10. What is the difference between the simple and compound interest of $300 for 3 yr. 3 mo. 3 da. at 3%? *Ans.* $1.03.

AVERAGING ACCOUNTS.

620. Averaging an Account is the process of finding the mean or equitable time for the payment of the balance of the account.

Averaging accounts is a case of Equation of Payments when there are both debits and credits in the account.

1. In the following account, required the balance and the time when due:

DR. JOSEPH ADAMS. CR.

1894.				1894.			
July 10	To merchandise,	300	00	July 20	By Cash,	150	00
Aug. 20	" "	400	00	Sept. 16	" "	300	00
Oct. 28	" "	600	00	Oct. 20	" "	500	00

OPERATION.

Due.	Time.	Items.	Products.	Due.	Time.	Items.	Products.
1894.				1894.			
July 10	00	$300	00000	July 20	10	$150	1500
Aug. 20	41	400	16400	Sept. 16	68	300	20400
Oct. 28	110	600	66000	Oct. 20	102	500	51000
		1300	82400			950	72900
		950	72900				
		350	9500	$9500 \div 350 = 27\frac{1}{7}$ da.			

SOLUTION.—Select the date of the item first due as the focal date, and find the time the others are due after it; then multiplying each item by the corresponding time, and taking the sums of the products, we find that if paid on the 10th of July the *Dr.* items must suffer a discount of $1 for 82400 days, and the *Cr.* items must suffer a discount of $1 for 72900 days. Subtracting the two sums, we find that the *Dr.* side must suffer a discount of $1 for 9500 days more than the *Cr.* side, hence $350, the balance of the items, must suffer a discount of $\frac{1}{350}$ of 9500 days, which is $27\frac{1}{7}$ days. Hence, the balance is due 27 days after July 10th, or on August 6th. Hence we have the following

Rule.—I. *Find when each item is due, take the earliest date as the focal date, find the difference between the focal date and the remaining dates, and multiply each item by the corresponding difference.*

II. *Balance the columns of products and also the columns of items, and divide the former by the latter; the quotient added to the focal date will give the equated time.*

III. *If the two balances be on opposite sides of the account, the quotient obtained must be subtracted from the focal date.*

1. Other dates than the earliest might be selected as the focal date. If we reckon from the last date, we have the interest instead of the discount.

2. Instead of *products* we may obtain the *interest* at any per cent. on each item, and divide the balance of interest by the interest on the balance of the account for one day; the quotient will be the number of days to be added to or subtracted from the focal date.

WRITTEN EXERCISES.

2. What is the balance of the following account, and when is it due? *Ans.* Balance, $750; due June 16th.

DR. WILLIAM MONROE. CR.

1893.			1893.		
June 8	To Mdse.,	$400	July 17	By Cash,	$200
July 10	" Sundries,	600	Aug. 12	" "	400
Aug. 15	" Mdse.,	450	Sept. 16	" Sundries,	100

3. What is the balance of the following account, and when is it due? *Ans.* Balance, $655; due April 20th.

DR. CHARLES JACKSON. CR.

1892.			1892.		
Jan. 1	To Sund. on 3 mo.	$575	Mar. 20	By Cash,	$200
Jan. 18	" " 3 mo.	480	Apr. 10	" "	450
Feb. 20	" " 3 mo.	800	Apr. 30	" Merchandise,	300
Mar. 16	" Cash,	250	May 12	" Cash,	500

SETTLEMENT OF ACCOUNTS.

621. An **Account Current** is a written statement of the debit and credit items of business transactions between two parties.

622. The **Adjustment** of an account is the determining of the balance due at a specified date.

623. An account is **Settled** upon *payment* of the adjusted balance or by *carrying* it to another account.

In finding the cash balance, interest should be allowed on each item for the time between the day it is due and the day of settlement.

Rule.—I. *Find the interest on each item from the time it becomes due to the date of settlement.*

II. *Add the interest to the item if due before the date of settlement, and subtract it when the item is due after the date of settlement. The difference of the sums of the results on both sides of the account will be the cash balance.*

1. An account may be *adjusted* by averaging it, and finding the amount of the balance from the time it becomes due till the time of settlement.

2. In *averaging* an account, we find *at what date the balance* is due; in *adjusting* an account, we find *what balance* is due at a *specified date.*

WRITTEN EXERCISES.

1. Required the cash balance of the following account, Dec. 12, 1894. interest at 6 per cent. *Ans.* 302.75\frac{5}{8}$.

GEORGE PATTERSON, IN ACCOUNT WITH TAYLOR & SON.

1894.				1894.			
June 7	To Mdse. on 3 mo.	750	00	Aug. 15	By Cash,	400	00
July 12	" " on 3 mo.	350	00	Sept. 18	" "	300	00
Sept. 5	" " on 3 mo.	500	00	Nov. 30	" "	600	00

2. Required the cash balance of the following account, Aug. 8, 1891, interest at 6 per cent. *Ans.* $279.29.

William White, in account with Henry Black.

1891.				1891.			
Mar. 16	To Mdse., 2 mo.	850	00	May 28	By Cash,	400	00
Apr. 20	" " 2 mo.	750	00	July 12	" "	800	00
May 24	" " 3 mo.	375	00	July 20	" "	300	00
June 12	" " 3 mo.	500	00	Aug. 4	" "	700	00

GEOMETRICAL METHOD OF EVOLUTION.

624. The **Geometrical Method** of explaining evolution is here given for teachers who may prefer it:

SQUARE ROOT.

1. Find the square root of 1225.

OPERATION.

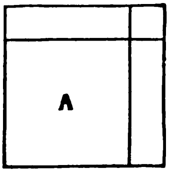
$1225(30$

$30^2 = 900 \quad 5$

$30 \times 2 = 60 \quad \overline{325}\ \overline{35}$

$(60+5) \times 5 = \underline{325}$

FIG. 1.



GEOMETRICAL SOL.—Let Fig. 1 represent a square which contains 1225 square units; then our object is to find the number of linear units in the edge. Since the square of a number consists of *twice as many places as the number itself, or twice as many less one,* the square root of 1225 will consist of two places, and hence will consist of tens and units.

The greatest number of tens whose square is contained in 1225 is 3 tens. Let A, Fig. 1, represent a square whose sides are 30 units; its area will be 30^2, or 900 square units. Subtracting 900 from 1225, we find remaining a surface containing 325 square units. By inspection we find this surface to consist principally of the two rectangles B and C, Fig. 2, each of which is 30 units long, and since they nearly complete the square, their area is nearly 325 units; hence if we divide 325 by their length, we will find their width. The length of both is $30 \times 2 = 60$; dividing 325 by 60, we find their width to be 5 units. Adding the length of the little corner square, Fig. 3, whose sides are 5 units, we find the entire length of the surface remaining after the removal of the square A is $60 + 5 = 65$ units, and, multiplying this by the width, we find the whole area of the remainder to be $65 \times 5 = 325$ square units. Subtracting 325 square units from the

FIG. 2.

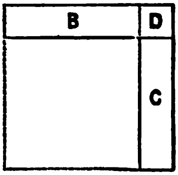
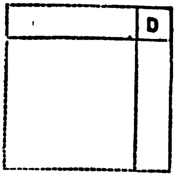


FIG. 3.



square units left after subtracting 900 square units, nothing remains; therefore the side of the square whose area is 1225 square units is 35 units; hence the square root of 1225 is 35.

NOTES.—1. When there are three figures in the root, after removing the first two rectangles and small square we have two rectangles and a small square remaining.

2. Square root can also be explained by building up the square instead of separating it into its parts.

CUBE ROOT.

2. Find the cube root of 91125.

GEOMETRICAL SOL.—Let Fig. 1 represent the cube which contains 91125 cubic units; then our object is to find the number of linear units in its edge. The number of terms in the root, found as before, is two. The greatest number of tens whose cube is contained in the given number is 4 tens. Let A, Fig. 1, represent a cube whose sides are 40; its contents will be $40^3 = 64000$. Subtracting 64000 from 91125, we find a remainder of 27125 cubic units, which, by removing the cube A from Fig. 1, leaves a solid represented by Fig. 3.

OPERATION.

$$\begin{array}{rr|l}
 & & 91125(40 \\
 & 40^3 = & \underline{64000}\ \ \underline{5} \\
 & & 27125\ \ 45 \\
3 \times 40^2 = & 4800 & \\
3 \times 40 \times 5 = & 600 & \\
5^2 = & \underline{25} & \\
 & \underline{5425} & \underline{27125}
\end{array}$$

FIG. 1.

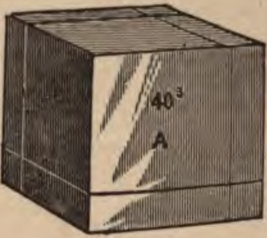


FIG. 2.

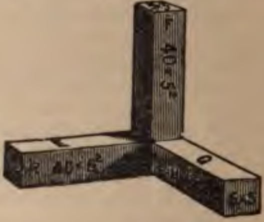


FIG. 3.

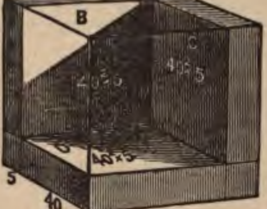
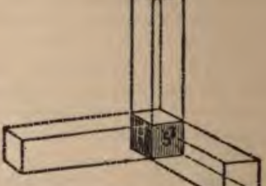


FIG. 4.



Inspecting this solid, we perceive that the greater part of it consists of the three rectangular slabs, B, C, and D, each of which is 40 units in length and breadth; hence, if we divide 27125 by the sum of the areas of one face of each, regarded as a base, we can ascertain their thickness. The area of a face of one slab is $40^2 = 1600$, and of the three, $3 \times 1600 = 4800$, and dividing 27125 by 4800, we have a quotient of 5; hence the thickness of the slab is 5 units.

Removing the rectangular slabs, there remain three other rectangular solids, E, F, G, as shown in Fig. 2, each of which is 40 units long and 5 units thick; hence the surface of a face of each is $40 \times 5 = 200$ square units, and of the three is $3 \times 40 \times 5 = 600$ square units.

Finally removing E, F, and G, there remains only the little corner cube H, Fig. 4, whose sides are 5 units and the surface of one of its faces $5^2 = 25$ square units. We now take the sum of the surfaces of the solids remaining after the removal of the cube A, and multiply this by the common thickness, which is 5, and we have their solid contents equal to $(4800 + 600 + 25) \times 5 = 27125$ cubic units, which, subtracted from the number of cubic units remaining after the removal of A, leaves no remainder. Hence the cube which contains 91125 cubic units is $40 + 5$, or 45 units on a side.

NOTE.—This can also be explained by building up the cube instead of separating it into its parts.

SHORT METHOD OF CUBE ROOT.

625. The abbreviation consists in obtaining the successive trial divisors by a law which enables us to use our previous work.

1. Extract the cube root of 14706125.

OPERATION.

```
                        14·706·125(245
                         8
12────────┐t. d.       ┌──────
 24       │──┐         │6706
  16──────┘  │         │
─────────────┤         │
1456         │ T. D.   │
  16─────────┘         │5824
─────────────────────────────────
1728─────┐ t. d.        │882125
  360    │              │
    25───┘              │
────────────────────────┤
176425 T. D.            │882125
──────────────────────────────────
```

SOLUTION.—We find, as before, the number of figures in the root, and the first term of the root, cube, subtract, and bring down the first period.

We then find, as before, the trial divisor, 12, by taking three times the square of the first term. Dividing, we find the second term of the root to be 4. We then, as before, take three times the product of the first and second terms, and the square of the second term, and add these to the trial divisor as a *correction* to obtain the *true divisor*, 1456. We then multiply 1456 by 4, and subtract and bring down the next period.

We then, to find the next *trial divisor*, take the *square* of the *last term*, which is 16, and add it to the previous *true divisor* and the *two corrections*, which were added to the previous trial divisor, and we have 1728 as the next trial divisor.

Then, to find the *true divisor*, we add three times the product of the last term of the root into the previous part of the root, and also the square of the last term, and have 176425 for the *true divisor*. Multiplying by 5, we have 882125.

The method is indicated in the following formula:

1. TRUE DIVISOR = TRIAL DIVISOR + PRODUCT + SQUARE.
2. TRIAL DIVISOR = SQUARE + TRUE DIVISOR + CORRECTIONS.

The method is readily explained either by the blocks or the algebraic formula.

WRITTEN EXERCISES.

Extract the cube root of

2. 12326391. *Ans.* 231.
3. 41063625. *Ans.* 345.
4. 1879080904. *Ans.* 1234.
5. 633839.779. *Ans.* 85.9.
6. 16348384872. *Ans.* 2538.
7. 16503.467336. *Ans.* 25.46.

ARITHMETICAL PROGRESSION.

626. An **Arithmetical Progression** is a series of numbers which vary by a common difference; as, 3, 5, 7, 9, etc.

627. The **Common Difference** is the difference between any two consecutive terms; thus, in the above series the common difference is 2.

628. The **Quantities** considered are five, any three of which being given, the others may be found.

QUANTITIES CONSIDERED.

1. The first term.
2. The last term.
3. The common difference.
4. The number of terms.
5. The sum of all the terms.

629. Given the first term, the common difference, and the number of terms, to find the last term.

1. The first term is 4, the common difference 2, and number of terms 12; required the last term.

SOLUTION.—The first term is 4, the second term equals 4 plus *once* the common difference, the third term equals 4 plus *twice* the common difference, etc.; hence the twelfth term equals the first term plus *eleven* times the common difference, which equals $4 + 2 \times 11 = 26$.

OPERATION.

2d = $4 + 2 = 6$
3d = $4 + 2 \times 2 = 8$
4th = $4 + 2 \times 3 = 10$
Hence 12th = $4 + 2 \times 11 = 26$

Rule. *The last term equals the first term, increased by the common difference multiplied by the number of terms less one.*

NOTE.—In a descending series we must subtract instead of adding.

WRITTEN EXERCISES.

2. Find the 10th term of the series 3, 6, 9, etc. *Ans.* 30.

3. Find the 20th term of the series 2, 5, 8, etc. *Ans.* 59.

4. Find the 24th term of the series 4, 8, 12, etc. *Ans.* 96.

5. Find the 22d term of the series 3, $5\frac{1}{3}$, $7\frac{2}{3}$, etc. *Ans.* 52.

6. Required the 57th term of a descending series, the 1st term being 70 and common difference $\frac{3}{4}$. *Ans.* 28.

7. A man bought 40 yards of muslin at $\frac{1}{2}$ cent for the first yard, 1 cent for the second, $1\frac{1}{2}$ for the third, and so on; what did the last yard cost? *Ans.* 20 cents.

630. Given the last term, the common difference, and the number of terms, to find the first term.

1. Required the first term, the last term being 24, the number of terms 12, and the common difference 2.

SOLUTION.—From the Rule in Case I. we have 24 = 1st term + 11 times 2; hence we find first term = 24 − 11 × 2 = 2.

OPERATION.

$24 = 1\text{st} + 2 \times 11$

$1\text{st} = 24 - 2 \times 11 = 2$

Rule.—*The first term equals the last term, diminished by the common difference multiplied by the number of terms less one.*

WRITTEN EXERCISES.

2. Required the first term, the last term being 85, common difference 5, and number of terms 16. *Ans.* 10.

3. A woman bought 40 yards of muslin at the rate of 20 cents for the last yard, $19\frac{1}{2}$ for the next to the last, and so on; what did the first yard cost? *Ans.* $\frac{1}{2}$ cent.

4. A man traveled for 12 days, traveling $1\frac{1}{2}$ miles farther each day, and on the last day he went 22 miles; how far did he travel the first day? *Ans.* $5\frac{1}{2}$ miles.

631. Given the first term, the last term, and the number of terms, to find the common difference.

1. What is the common difference if the first term is 4, the last 76, and the number of terms 25?

SOLUTION.—By Case I. we have 76 = 4 + (24 times the *common difference*); hence, the *common difference* equals (76 − 4) ÷ 24, which equals 3.

OPERATION.

$76 = 4 + 24 \times \text{diff.}$

$$\text{diff.} = \frac{76 - 4}{24} = 3$$

Rule.—*The common difference equals the difference of the extremes divided by the number of terms less one.*

WRITTEN EXERCISES.

2. What is the common difference if the first term is 4, the last 400, and the number of terms 100? *Ans.* 4.

3. The amount of $100 at 6% for 20 years is $220; what is the annual interest? *Ans.* $6.

4. The youngest of 11 children is 62 and the oldest 82 years old, their ages being in arithmetical progression; what is the common difference of their ages? *Ans.* 2 years.

632. Given the first term, the last term, and the common difference, to find the number of terms.

1. What is the number of terms, if the first term is 76, the last term 4, and the common difference 3?

Solution.—By Case II. we have 76 = 4 + (No. of terms − 1) × 3; from which we have (No. of terms − 1) × 3 = 76 − 4, and No. of terms − 1 = (76 − 4) ÷ 3, or No. of terms = (76 − 4) ÷ 3 + 1 = 25.

OPERATION.

$$76 = 4 + (n-1) \times 3$$
$$(n-1) \times 3 = 76 - 4$$
$$n = \frac{76-4}{3} + 1 = 25$$

Rule.—*The number of terms equals the difference between the extremes divided by the common difference, plus one.*

WRITTEN EXERCISES.

2. What is the number of terms if the first term is 4, the last 400, and the common difference 4? *Ans.* 100.

3. In what time will $100 at 6 per cent. simple interest amount to $220? *Ans.* 20 years.

4. A laborer received 50 cents the first day, 60 cents the second, and so on till he received $2.80 a day; how many days did he work? *Ans.* 24 days.

633. Given the first term, the last term, and the number of terms, to find the sum of the series.

1. Given the first term 3, the last term 19, and the number of terms 5, to find the sum of the terms.

Solution.—To derive the rule, we find by Case III. the common difference to be 4. Writing the series in its natural, and then in an inverted, order, we take the sum of the two series, and we have *twice the sum*, equal to 22 taken 5 times —that is, (3 + 19) × 5; hence, the

OPERATION.

$$\begin{aligned} \text{Sum} &= 3 + 7 + 11 + 15 + 19 \\ \text{Sum} &= 19 + 15 + 11 + 7 + 3 \\ \hline 2 \times \text{Sum} &= 22 + 22 + 22 + 22 + 22 \\ 2 \times \text{Sum} &= 22 \times 5 = (3 + 19) \times 5 \\ \text{Sum} &= \frac{3+19}{2} \times 5 = 55 \end{aligned}$$

sum equals $\frac{1}{2}$ of $(3+19)\times 5$, which equals 55. Now, $(3+19)$ is the *sum of the extremes*, and 5 is the *number of terms;* hence we have the following

Rule.—*The sum of an arithmetical series equals half the sum of the extremes multiplied by the number of terms.*

WRITTEN EXERCISES.

Find the sum

2. Of 12 terms of the series 2, 4, 6, etc. *Ans.* 156.

3. Of 32 terms of the series 3, 6, 9, etc. *Ans.* 1584.

4. Of 40 terms of the series 4, 7, 10, etc. *Ans.* 2500.

5. Of 45 terms of the series 2, $5\frac{1}{4}$, $8\frac{1}{2}$, etc. *Ans.* $3307\frac{1}{2}$.

6. How far can I walk in 10 days, going 15 miles the first day, and increasing the rate 5 miles a day? *Ans.* 375 mi.

7. 100 oranges are placed in a row 2 yards apart, the first being 2 yards from a basket; how far will a boy travel, starting from the basket, to gather them singly into the basket?
Ans. 11 mi. 152 rd. 4 yd.

8. A body will fall $16\frac{1}{12}$ feet in 1 second, 3 times as far the next, 5 times as far the third, etc.; how far will it fall in a minute? *Ans.* 10 mi. $309\frac{1}{11}$ rd.

GEOMETRICAL PROGRESSION.

634. A **Geometrical Progression** is a series of numbers which vary by a common multiplier; as, 2, 6, 18, 54, etc.

635. The **Rate**, or *Ratio*, is the common multiplier; thus, in the above series, the rate is 3.

636. In an **Ascending** series the rate is greater than a unit; in a **Descending** series the rate is less than a unit.

637. The **Quantities** considered are five, any three of which being given, the others may be found.

QUANTITIES CONSIDERED.

1. The first term.
2. The last term.
3. The number of terms.
4. The rate.
5. The sum of the terms.

638. Given the first term, the rate, and the number of terms, to find the last term.

1. The first term equals 2, the rate 4, and the number of terms 6; required the last term.

SOLUTION.—The 2d term equals 2×4; the 3d term equals 2×4 multiplied by 4, or 2×4^2, which is the 1st term into the second power of the rate; the 4th term equals 2×4^2 multiplied by 4, or 2×4^3, which is the 1st term into the 3d power of the rate; hence, the 6th term equals the first term into the 5th power of the rate, or 2×4^5, which equals 1024. Hence

OPERATION.

$$\begin{aligned} 2\text{d} &= 2 \times 4 \\ 3\text{d} &= 2 \times 4^2 \\ 4\text{th} &= 2 \times 4^3 \\ \text{hence } 6\text{th} &= 2 \times 4^5 = 1024 \end{aligned}$$

Rule.—*The last term equals the first term multiplied by the rate raised to a power one less than the number of terms.*

NOTE.—The table of compound interest, page 415, can be derived from this case, 1 plus the rate per cent. being the *rate.*

WRITTEN EXERCISES.

2. Find the 8th term of the series 3, 6, 12. *Ans.* 384.

3. Find the 9th term of the series 4, 8, 16, etc. *Ans.* 1024.

4. Find the 10th term of the series 1, $\frac{1}{2}$, $\frac{1}{4}$, etc. *Ans.* $\frac{1}{512}$.

5. Find the 10th term of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, etc. *Ans.* $\frac{1}{19683}$.

6. Find the 12th term of the series 24, 12, 6, etc. *Ans.* $\frac{3}{256}$.

7. Find the 13th term of the series, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, etc. *Ans.* $\frac{2048}{531441}$.

8. The first term of a descending series is 24 and the rate is $\frac{1}{2}$; what is the 10th term? *Ans.* $\frac{3}{64}$.

9. A merchant doubles his capital every 4 yr.; if he begins with $1000, how much has he at the end of 20 yr.? *Ans.* $32,000.

10. A man bought 20 cows, agreeing to pay for them all as much as the last cow would cost, at the rate of 1 cent for the first, 2 cents for the second, 4 cents for the third, etc.; what did they cost? *Ans.* $5242.88.

639. Given the first term, the rate, and the last term or number of terms, to find the sum of the terms.

1. The first term is 2, the rate is 4, and number of terms 6; required the sum of the terms.

OPERATION.

$$\begin{array}{rl} \text{Sum} = & 2 + 8 + 32 + 128 + 512 + 2048 \\ \text{Sum} \times 4 = & \phantom{2 + {}} 8 + 32 + 128 + 512 + 2048 + 8192 \\ \hline 3 \times \text{the sum} = & 8192 - 2 \\ \text{Sum} = & \dfrac{8192 - 2}{3} = 2730, \textit{ Ans.} \end{array}$$

SOLUTION.—Writing the series expressing the sum, and then multiplying by the rate, and taking the difference of the two series, we have 3 *times* the *sum* equals $8192 - 2$, and the sum equals $\frac{1}{3}$ of $8192 - 2$, which is 2730. In this solution we observe that 8192 is the last term multiplied by the rate, and that this is diminished by the first term, and the difference divided by the rate minus one; hence we have the following

Rule.—*To find the sum, multiply the last term by the rate, subtract the first term, and divide the remainder by the rate diminished by unity.*

NOTE.—In a decreasing series we subtract the last term multiplied by the rate from the first term, and divide by 1 minus the rate.

WRITTEN EXERCISES.

Find the sum

2. Of 2, 4, 8, etc. to 8 terms. *Ans.* 510.

3. Of 1, 3, 9, etc. to 9 terms. *Ans.* 9841.

4. Of 3, 6, 12, etc. to 10 terms. *Ans.* 3069.

5. Of 16, 8, 4, etc. to 7 terms. *Ans.* $31\frac{3}{4}$.

6. Of $3\frac{3}{4}$, $\frac{3}{2}$, $\frac{3}{5}$, etc. to 6 terms. *Ans.* $6\frac{561}{2500}$.

7. A mother gave her daughter 1 cent at birth, doubling it on each succeeding birthday; how much was the daughter worth when she became 21 years of age? *Ans.* $20971.51.

8. A lady, thinking $2 a yard too much for a silk dress containing 15 yards, agreed to pay 1 cent for the first yard, 2 cents for the second, etc.; which price was the greater, and how much? *Ans.* 2d, $297.67 more.

9. A man wishing to buy a horse refused to give $250, but agreed to pay 1¢ for the first nail in his shoes, 2¢ for the 2d, 4¢ for the third, etc.; what did the horse cost, there being 32 nails in his shoes? *Ans.* $42949672.95.

INFINITE SERIES.

640. An **Infinite Series** is a series in which the number of terms is unlimited.

641. In a descending infinite series we may consider the last term to be zero; hence the above rule becomes

Rule.—*The sum of an infinite series equals the first term, divided by a unit minus the rate.*

1. What is the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.?

SOLUTION.—In this series the first term is 1, and the rate $\frac{1}{2}$, and the last term is regarded as zero; hence, we have the sum of the series equal to 1 divided by $1 - \frac{1}{2}$, or $1 \div \frac{1}{2}$, which is 2.

OPERATION.

$$\text{Sum} = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2, \textit{ Ans.}$$

WRITTEN EXERCISES.

2. Sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc.? *Ans.* 1.

3. Sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, etc.? *Ans.* $\frac{1}{2}$.

4. Sum of the infinite series $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc.? *Ans.* $\frac{1}{2}$.

5. A ball falls 8 ft. to the floor and bounds back 4 ft., then falling bounds back 2 ft., and so on; how far will it move before coming to rest? *Ans.* 24 feet.

6. A hound and fox, 10 rods apart, run so that when the hound runs 10 rods the fox runs 1 rod, etc.; how far will the hound run to catch the fox? *Ans.* 11$\frac{1}{9}$ rods.

THE METRIC SYSTEM.

642. THE **Metric System** of weights and measures is used by most civilized nations except the United States and Great Britain.

The old system of weights and measures in our country is irregular, difficult to learn, and inconvenient to apply. The same is true with the old systems of all nations. Originating by chance, rather than by science, they lacked the simplicity of law, and were, therefore, irregular and chaotic.

In 1795, France adopted a system of weights and measures, called the Metric System, based upon the decimal method of notation, all the divisions and multiples being by 10. It was regarded as so great an improvement upon the old methods that it has since been introduced into Spain, Belgium, Portugal, Switzerland, Holland, Italy, Germany, Austria, Sweden, Denmark, Greece, Mexico, Brazil, and by most of the South American States, and in the most of these countries its use is compulsory. In 1864 the British Parliament passed an act permitting its use throughout the empire whenever parties should agree to use it.

The introduction of the Metric System into this country had been long recommended by scientific men and by such statesmen as Madison, Jefferson, John Quincy Adams, etc. In 1866, through the influence of Charles Sumner, Congress authorized its use in the United States, and provided for its introduction into the post-offices for the weighing of letters and papers. To facilitate its adoption, a convenient standard of comparison was furnished, by making the new five-cent piece five grams in weight and one-fiftieth of a meter, or two centimeters, in diameter.

The *advantages* of the Metric System are numerous and important:

1. It is easily learned; a school-boy can learn it in a single afternoon.

2. It is easily applied, all the operations being the same as in simple numbers.

3. It does away with addition, subtraction, multiplication, division, and reduction of compound numbers.

4. It will facilitate commerce, giving the nations a universal system of weights and measures.

643. The **Metric System** of weights and measures is based upon the decimal system of notation.

644. In this system we first establish the unit of each measure, and then derive the other denominations by taking decimal multiples and divisions of the unit.

645. Names.—We first name the unit of any measure, and then derive the other denominations by adding prefixes to the unit name.

646. The *higher denominations* are expressed by prefixing to the name of the unit

Deka,	**Hekto,**	**Kilo,**	**Myria.**
10	100	1000	10,000

The *lower denominations* are expressed by prefixing to the name of unit

Deci,	**Centi,**	**Milli.**
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

647. Units.—The following are the different units, with their English pronunciation:

Measure.	Unit.	Pronunciation.	Measure.	Unit.	Pronunciation.
LENGTH,	**Meter**	(meter).	CAPACITY,	**Liter**	(leeter).
SURFACE,	**Are**	(air).	WEIGHT,	**Gram**	(gram).
VOLUME,	**Stere**	(stair).	VALUE,	**Dollar.**	

1. Many of the denominations given in the system are not generally used. Those in common use are printed in bold-faced type.

2. The fractional parts of the standard units are indicated by abbreviations beginning with a *small letter;* the multiples of the unit by those beginning with a *capital.*

MEASURES OF LENGTH.

648. The **Meter** is the *unit of length.* It is the ten-millionth part of the distance from the equator to the poles, and equals 39.37 inches, or 3.28 feet.

TABLE.

10 **Millimeters** (^{mm}) = 1 **Centimeter**, ^{cm} = .3937079 in.
10 Centimeters = 1 Decimeter, ^{dm} = 3.937079 in.
10 Decimeters = 1 **Meter**, ^{m} = 39.37079 in.
10 Meters = 1 Dekameter, ^{Dm} = 32.80899 ft.
10 Dekameters = 1 Hektometer, ^{Hm} = 19.88423 rd.
10 Hektometers = 1 **Kilometer**, ^{Km} = .621382 mi.
10 Kilometers = 1 Myriameter, ^{Mm} = 6.21382 mi.

1. The *meter* is very nearly 3 feet 3 inches and 3 eighths of an inch in length, which may be easily remembered as the *rule of three threes.*

2. Cloth, etc. are measured by the *meter;* very small distances, by the *millimeter;* great distances, by the *kilometer.*

3. The 5-cent piece of 1866 is very nearly $\frac{1}{50}$ of a *meter* in diameter; hence, its diameter is about $\frac{1}{5}$ of a *decimeter*, or 2 *centimeters.* It was ordered to be $\frac{1}{50}$ of a meter in diameter, but owing to the composition of the alloy it was necessary to make its diameter a little greater; 48.6 nickel 5-cent pieces laid side by side measure one meter.

4. A *decimeter* is about 4 inches; a *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile; a *millimeter*, about $\frac{1}{25}$ of an inch. The *inch* is about $2\frac{1}{2}$ centimeters; the *foot*, 3 decimeters; the *rod*, 5 meters; the *mile*, 1600 meters or 16 hektometers.

MEASURES OF SURFACE.

649. The **Are** is the *unit of surface* used to measure land. The *are* is a *square dekameter.* It equals 119.6 sq. yd., or 0.0247 acre.

TABLE.

10 Milliares (^{ma}) = 1 **Centiare**, ^{ca} = 1.196 sq. yd.
10 Centiares = 1 Deciare, ^{da} = 11.960 sq yd.
10 Deciares = 1 **Are**, ^{A} = 119.6034 sq. yd.
10 Ares = 1 Dekare, ^{Da} = .24714 A.
10 Dekares = 1 **Hektare**, ^{Ha} = 2.4714 A.
10 Hektares = 1 Kilare, ^{Ka} = 24.714 A.
10 Kilares = 1 Myriare, ^{Ma}. = 247.14 A.

1. The *are, centiare*, and *hektare* are the denominations principally used, as these are exact squares. The *centiare* is a square whose side is 1 meter; the *hektare* is a square whose side is 100 meters.

The *are* = 100 square meters. The *centiare* = 1 square meter.
The *hektare* = 10,000 square meters.

2. The *deciare* is not a *square*—it is merely the tenth of an are; the *dekare* is not a square—it is merely ten ares.

3. A *hektare* equals nearly $2\frac{1}{2}$ acres; a *centiare* equals nearly $1\frac{1}{5}$ sq. yd. *An acre is* very nearly 40 ares.

MEASURES OF OTHER SURFACES.

650. All surfaces besides land are measured by the *square meter, square decimeter*, etc. The measures are shown by the following table:

TABLE.

100 Sq. millimeters ($^{\text{sq mm}}$) = 1 **Sq. centimeter,** $^{\text{sq cm}}$ = .155 + sq. in.
100 Sq. centimeters = 1 Sq. decimeter, $^{\text{sq dm}}$ = 15.5 + sq. in.
100 Sq. decimeters = 1 **Sq. meter,** $^{\text{sq m}}$ = 1.196 + sq. yd.

The sq. kilometer = 247.114 A. = .3861 sq. mi. is also used.

MEASURES OF VOLUMES.

651. The units of volume are the *cubic meter* and its divisions. The measures are shown by the following table:

TABLE.

1000 Cubic millimeters ($^{\text{cu mm}}$) = 1 **Cubic centimeter,** $^{\text{cu cm}}$.
1000 Cubic centimeters = 1 Cubic decimeter, $^{\text{cu dm}}$.
1000 Cubic decimeters = 1 **Cubic meter,** $^{\text{cu m}}$.

In measuring wood the cubic meter is called a *stere.* 3.6 *steres,* or 36 *decisteres,* very nearly equal the common *cord.*

MEASURES OF CAPACITY.

652. The **Liter** is the *unit of capacity.* It equals a *cubic decimeter;* that is, a cubic vessel whose edge is one-tenth of a meter.

653. This measure is used for measuring liquids and dry substances. The *liter* is a cylinder, and holds 2.1135 pints wine measure, or 1.816 pints dry measure.

TABLE.

10 Milliliters ($^{\text{ml}}$) = 1 **Centiliter,** $^{\text{cl}}$ = .61027 cu. in.
10 Centiliters = 1 Deciliter, $^{\text{dl}}$ = 6.1027 cu. in., or .845 gi.
10 Deciliters = 1 **Liter,** $^{\text{l}}$ = .908 qt., or 1.0567 qt.
10 Liters = 1 Dekaliter, $^{\text{Dl}}$ = 9.08 qt., or 10.567 qt.
10 Dekaliters = 1 **Hektoliter,** $^{\text{Hl}}$ = 2.8375 bu., or 26.418 gal.
10 Hektoliters = 1 Kiloliter, $^{\text{Kl}}$ = 28.375 bu., or 264.18 gal.

1. The *liter* is principally used in measuring *liquids,* and the *hektoliter* in measuring grains, etc.

2. The *liter* equals nearly $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, or nearly $\frac{1}{35}$ of a bushel measure.

3. The *hektoliter* is about $2\frac{5}{6}$ bushels, or $\frac{5}{6}$ of a barrel. 4 *liters* are a little more than a *gallon*; 35 *liters*, very nearly a *bushel*.

MEASURES OF WEIGHT.

654. The **Gram** is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of melting ice. The *gram* equals 15.432 Troy grains.

TABLE.

10 **Milligrams** (^{mg})	=1 **Centigram**, ^{cg}	=.15432+ gr.
10 Centigrams	=1 Decigram, ^{dg}	=1.54324+ gr.
10 Decigrams	=1 **Gram**, ^{g}	=15.43248+ gr.
10 Grams	=1 Dekagram, ^{Dg}	=.35273 + oz.
10 Dekagrams	=1 Hektogram, ^{Hg}	=3.52739+ oz.
10 Hektograms	=1 **Kilogram**, ^{Kg}	=2.20462+ lb.
10 Kilograms	=1 Myriagram, ^{Mg}	=22.04621+ lb.

1. The *gram* is used in weighing letters and mixing and compounding medicines, and in weighing all very light articles. The new 5-cent coin (dated 1866) weighs 5 grams.

2. The *kilogram* is the ordinary unit of weight, and is generally abbreviated into *kilo.* It equals about $2\frac{1}{5}$ pounds avoirdupois. Meat, sugar, etc. are bought and sold by the *kilogram.*

3. In weighing heavy articles two other weights—the *quintal* (100 kilograms) and the *tonneau* (1000 kilograms)—are used. The *tonneau* is between our *short ton* and *long ton.*

4. The *avoirdupois ounce* is about 28 *grams*; the *pound* is a little less than $\frac{1}{2}$ a *kilo.*

WRITTEN EXERCISES.

1. If a letter weighs 2.5 grams, how many such letters will it take to weigh a kilogram? *Ans.* 400 letters.

2. A lady bought 11.5 meters of silk for a dress, at the rate of \$4.75 a meter; what did it cost her? *Ans.* \$54.625.

3. My butcher's bill one month was 87.5 kilograms of beef, at $18\frac{3}{4}$ cents a kilo; what was the bill? *Ans.* \$16.40$\frac{5}{8}$.

4. How much must I pay for 56.25 liters of coal oil, at the rate *of* $18\frac{3}{4}$ *cents* a liter? *Ans.* \$10.546+.

5. A kilogram weighs 2.2046 lb.; what is the weight of 56½ tonneaux? *Ans.* 124559.9 lb.

6. A bought 2500 ares of land, at $4.50 an are, and sold it for $525 a hektare; what was the gain? *Ans.* $1875.

7. If 15 steres of wood cost $22.50, what must I pay for 24.5 steres at the same rate? *Ans.* $36.75.

8. If a kilogram of sugar is worth 21¾ cents, how many kilos can I buy for $100? *Ans.* 459.77 + kilos.

9. The height of a pole is 68.325^{m}; how long would it take a worm to climb to its top, at the rate of 15^{m} a day? *Ans.* 4.555 da.

10. A kilometer is about ⅝ of a mile; how many kilometers from Lancaster to Philadelphia, 70 miles? *Ans.* 112^{Km}.

11. How much must I pay for $23\frac{3}{4}^{m}$ of silk, at 8 francs 25 centimes a meter? *Ans.* 195.94 – fr.

12. What cost 3 kilares, 7 hektares, 6 deciares of land, at $275.25 a hektare? *Ans.* $10185.90.

13. It is about 100 miles from Philadelphia to New York; how many kilometers is it? *Ans.* 160^{Km}.

14. How much will it cost to excavate $12\frac{4}{5}^{cu\ m}$ of earth, at $37.25 a cubic meter? *Ans.* $476.80.

15. What is the width of the Atlantic in kilometers, the width being about 3000 miles? *Ans.* 4800^{Km}.

16. What must I pay for 25^{s}, 2^{ds}, and 5^{cs} of wood, at the rate of $2.65 a stere? *Ans.* $66.91¼.

17. How long will it take a man to walk from Philadelphia to New York, at the rate of 8^{Km} an hour? *Ans.* 20 hours.

18. Two vessels are 432^{Km} apart, and sail toward each other, each at the rate of 18^{Km} an hour; in how many hours will they be together? *Ans.* 12 hours.

19. A block 3.5^{m} long, $.75^{m}$ wide, and $.8^{m}$ thick cost $12; what would a cubic meter of marble cost at the same rate? *Ans.* $5.71 +.

20. A man bought 7000^{g} of jewels, at 40 francs a gram, and sold them at $15 a pennyweight; how much was gained or lost? *Ans.* $13475.

PROBLEMS ON IMPORTS.

1. An importer bought 428.5^{m} of silk in France, at 18 francs a meter, sent it to the United States, paying 25 cents a meter shipping and duty, and sold it for \$5.25 a meter; what was his gain? *Ans.* \$653.89.

2. A man bought a valuable gem in France which weighed 325.75^{g}, @ 10.25 francs; the duty on it was \$6.25; how must he sell it a gram to clear \$150? *Ans.* \$2.46.

3. I bought 125.75^{l} of wine in France, at 45.25 francs a liter, paid \$1.25 a liter duty and freight, and sold it at \$12.50 a liter; how much did I gain? *Ans.* \$316.48.

4. An importer bought 625.5^{l} of French brandy, at 7.55 francs a liter, paid 15 cents a liter duty and freight, and sold it in New York at \$1.65 a liter; how much did he gain? *Ans.* \$26.80.

5. A man bought 200^{m} of cloth in France, at 16.25 francs a meter; he paid $12\frac{1}{2}$ cents a yard duty and freight, and sold it in Boston at \$4.62$\frac{1}{2}$ a yard; what was the gain? *Ans.* \$357.

6. An importer bought 480^{g} of jewels, at 12.25 francs a gram, paid \$5.25 an ounce shipment and duty, and sold them in Philadelphia at \$102.75 an ounce; what was the gain? *Ans.* \$369.78.

7. A wine-merchant bought 180^{l} of brandy in Havre, at $32\frac{1}{2}$ decimes a liter; he paid $2\frac{1}{2}$ decimes a liter shipment and \$2.25 a gallon duty, and sold it in New York at \$6.75 a gallon; what was his gain? *Ans.* \$92.40.

NOTES ON TABLES OF WEIGHTS AND MEASURES.

655. The following notes on the Tables of Weights and Measures will be found of interest:

TROY WEIGHT.

I. NAME.—The term *Troy* is said to be derived from *Troyes*, the name of a town in France where this weight was first used in Europe. It was brought from Cairo in Egypt during the Crusades of the 12th century.

II. TERMS.—The term *pound* is from the Latin *pendo*, to *bend* or *weigh*. The term *ounce* is from the Latin *uncia*, a *twelfth part*, the ounce being one-twelfth part of a pound. The *pennyweight* was the weight of the old English silver penny. The term *grain* originated in the custom of using a number of grains of wheat for the weight of a penny. These grains were taken from the middle of the ear and well dried, thirty-two at first, and afterward twenty-four, being used to make a pennyweight.

III. Symbols.—The symbol *oz.* is thought to be from the Spanish word *onza*, signifying *ounce*, though Webster derives it from the use of the termination ʒ to express abbreviation, which was afterward changed to *z*; *lb.* is from *libra*, the Latin for pound. *Pwt.* is a combination of *p.* for *penny* and *wt.* for *weight.*

APOTHECARIES' WEIGHT.

I. Name.—The name arises from the weight being used by ***apothecaries.***

II. Terms.—The term *scruple* is from the Latin *scrupulus*, a *little stone.* The term *dram* is from the Greek *drachma, a piece of money.*

III. Symbols.—The symbols have been supposed to be modifications of the figure 3, suggested by there being 3 scruples in a dram. Another supposition is that they are from inscriptions upon the ancient monuments of Egypt.

IV. Physicians use the Roman notation in writing prescriptions, using the small letters, preceded by the symbols, and writing *j* for *i* when it terminates a number. Thus, 12 gr. is written gr. xij.; 2 scruples, ℈ij. ℞ is an abbreviation for *recipe, take; ā* or *āā* (from the Greek ἀνά) means, *of each; ss.* for *semis*, or *half*, as, ℈ivss. means 4½ scruples; *P.* for *particula*, or *little part; P. æq.* for *equal parts; q. p., quantum placet, as much as you please.*

AVOIRDUPOIS WEIGHT.

I. Name.—The term *Avoirdupois* is probably from the French *avoir du poids, to have weight.*

II. Terms.—The term *ton* is from the Saxon *tunne*, a *cask.* The origin of the other terms has already been given. The symbol *cwt.* is from *centum, hundred*, and *weight.* The term *dram* has been used for $\frac{1}{16}$ of an ounce, but is obsolete, fractions of an ounce being used in business transactions.

IV. In Great Britain 28 lb. equal 1 qr., 112 lb. equal 1 cwt., and 2240 lb. equal 1 ton. These are called the *long hundred* and *long ton;* they were formerly used in this country, but are now used only at the custom-houses in invoices of English goods, in ocean freights, in the wholesale iron and plate trade, and in wholesaling and freighting coal from the coal-mines of Pennsylvania.

V. The following denominations are frequently used:

6½ lb. of petroleum make 1 gallon.
25 " powder " 1 barrel.
56 " butter " 1 firkin.
84 " " " 1 tub.
100 " grain or flour " 1 cental.
100 " dry fish " 1 quintal.
100 " nails " 1 keg.

100 lb. of raisins make 1 cask.
196 " flour " 1 barrel.
200 " pork, beef, or fish, 1 barrel.
240 " lime " 1 cask.
280 " salt at N. Y. S. works make 1 barrel.
600 " rice make 1 barrel.

LONG MEASURE.

I. TERMS.—The term *inch* is from *uncia*, a *twelfth; foot* is from the human foot; *yard* was a *rod* or *shoot; rod* is from a *measuring-stick* or *rod; furlong*, which has now become obsolete, is from *fur*, *furrow*, and *lang*, *long*, the *length of a furrow; mile* is from *mille passuum*, 1000 paces; *span* is the space measured from the end of the thumb to the end of the little finger extended; *cubit*, from the elbow to the end of the middle finger; *fathom*, the length of the two arms extended. The ancient yard of England is said to have been determined by the length of the arm of King Henry I.

II. UNIT.—The *standard unit* of length is the *yard*, from which all other measures of length, and also those of capacity, weight, etc.. are derived. It is identical with the Imperial yard of Great Britain, which, under William IV., was declared to be fixed by dividing a pendulum, which vibrates seconds in a vacuum at the level of the sea, at 62° Fahr., in the latitude of London, into 391393 equal parts, and taking 360000 of these parts for the yard. Subsequent scientific experiments have proved that such a standard is impracticable.—*Brooks's Philosophy of Arithmetic.*

III. THE MILE.—The *geographic* or *nautical mile* is equal to 1 minute of one of the great circles of the earth; hence it equals $\frac{1}{60}$ of $\frac{1}{360}$ of the circumference of the earth, which equals about 1.15 statute miles. The English mile is the same as that of the United States. 3 statute miles make a *land league;* 3 nautical miles make a *nautical league.*

IV. OTHER MEASURES.—The following denominations are frequently used: In clockmaking, 6 *points* = 1 *line* and 12 *lines* = 1 *inch;* in measuring the foot, 3 *barleycorns* or *sizes* = 1 inch; in measuring the height of horses, 4 inches = 1 *hand*, the measure being taken directly over the fore shoulder; 1 *span* = 9 inches; 1 *common cubit* = 18 inches, and 1 *sacred cubit* = 21.888 inches; 1 *pace* = 3.3 feet; a *knot* is equal to a nautical mile. Formerly 40 rods made 1 *furlong*, and 8 furlongs 1 mile, but these are seldom used.

SURVEYORS' LINEAR MEASURE.

I. NAME.—*Gunter's chain* is named after the reputed inventor, Edmund Gunter, an English mathematician, born 1581.

II. The denomination *rods* is seldom used by surveyors, distances being represented in chains and links. Since each link is $\frac{1}{100}$ of a chain, the number of links is generally expressed as a decimal; thus, 5 chains and 47 links are written 5.47 chains. Engineers generally use a chain 100 feet long.

SURFACE OR SQUARE MEASURE.

I. TERMS.—*Perch* is from the French *perche*, a pole; *acre* was primarily an open ploughed or sowed field.

II. In the parts of the Mississippi Valley settled by the French the *old French arpent* ($\frac{5}{6}$ of an acre) is still used as the unit of measure.

LIQUID OR WINE MEASURE.

I. Name.—It is called *Wine Measure* because wine was measured by it, while beer was measured by another measure.

II. Terms.—*Gill* is from Low Latin *gilla, a drinking-glass; pint* is from the Anglo-Saxon *pyndan, to shut in, to pen,* or from the Greek *pinto, to drink; quart* is from the Latin *quartus,* a *fourth.* The derivation of *gallon* is not clear; in the French a *galon* is a *grocer's box.*

III. A pint of water weighs nearly one pound; hence the old adage, "A pint's a pound, the world around."

IV. Ale, beer, and milk were formerly sold by a *gallon* of 282 cu. in., the subdivisions being *quarts* and *pints.* The measure was greater than wine measure, as beer was less costly than wine. This measure is now seldom used.

V. The Imperial gallon of Great Britain contains 277.274 cu. in., and is equal to 1.2 United States gallon.

APOTHECARIES' FLUID MEASURE.

I. Terms.—*Minim* is from the Latin *minimus, the least,* the minim being the smallest fluid measure used. Several of the other terms are formed by prefixing *fluid* to the terms of Apothecaries' Weight.

II. Symbols.—*Cong.* is the abbreviation of *congius,* the Latin for gallon. *O.* is the initial of *octarius,* the Latin for *one-eighth,* the pint being one-eighth of a gallon. Drops are indicated in a physician's prescription by *gtt.,* for the Latin *guttæ.*

DRY MEASURE.

I. *Peck* is supposed to be a corruption of *pack,* or to be derived from the French *picotin,* a *peck.*

II. The *Chaldron,* consisting in some places of 36 bu. and in others of 32 bu., is used in some parts of the United States for measuring coal, but is being discontinued here, as it has been in England. One-half of a peck, or 4 quarts, is called a *dry gallon.*

In buying and selling grain and seeds, etc. the Boards of Trade of the principal cities of the United States use the number of pounds given in the following table as the equivalent of a bushel:

TABLE OF POUNDS IN A BUSHEL.

Barley	48	Corn, shelled	56	Peas	60
Beans	60	Corn in the ear	70	Rye	56
Buckwheat	48	Malt	34	Timothy-seed	45
Clover-seed	60	Oats	32	Wheat	60

TIME MEASURE.

Terms.—*Second* and *minute* are parts of an hour, corresponding to the parts of a degree in Circular Measure. *Hour* is derived from the Latin *hora,* originally a definite space of time fixed by natural laws; a *day,* derived from the Saxon *daeg*, is the time of the revolution of the earth upon its axis; a *week* is a period of uncertain origin, but which has been used from time immemorial in Eastern countries; a *month*, from Saxon *monadh*, from *mona*, the moon, is the time of one revolution of the moon around the earth; a *year*, from the Saxon *gear*, is the time of the earth's revolution around the sun; a *century* comes from the Latin *centuria*, a collection of a hundred things.

THE CALENDAR.

656. The time from any day of one month to any day of another month in the same year is readily found by the following table:

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January . . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	335	365	31	61
November . .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

METHOD OF USING THE TABLE.—Suppose we wish to find the number of days from March 10th to November 16th. We find March in the first column, and November at the top, and at the intersection we find 245, to which adding 6 days we have 251, the number of days required. The table being constructed for February 28 days, the proper allowance must be made for leap year.

TERMS.—January is derived from *Janus*, the god of the year, to whom this month was sacred. February is from *februa*, the Roman festival of expiation, celebrated on the 15th of this month. January and February were added to the Roman calendar by Numa, Romulus having previously divided the year into 10 months. March is from *Mars*, the god of war and reputed father of Romulus. It was the first month of the Roman calendar. April is probably from the Latin *aperire*, to open, from the opening of the buds or the bosom of the earth in producing vegetation. May is from *Maia*, the mother of Mercury, to whom the Romans offered sacrifices on the first day of this month. June is from *Juno*, the sister and wife of Jupiter, to whom it was sacred. July was named by Mark Antony after *Julius* Cæsar, who was born in this month. It was previously called Quintilis. August was named after *Augustus* Cæsar, who entered upon his first consulate in this month. It was formerly called Sextilis, or sixth month. September, October, November, and December are respectively named from the Latin numerals, *Septem*, *Octo*, *Novem*, and *Decem*, as, when the year began in March, they were the seventh, eighth, ninth, and tenth months, as their names indicate.

ADJUSTMENT OF THE CALENDAR.

The Rule for Leap Year given on page 177 is explained as follows:

EXPLANATION.—I. Since each year consists of 365 days 5 h. 48 min. 49.7 sec., if we reckon 365 days as one year, the time lost in the calendar in one year is 5 h. 48 min. 49.7 sec., and the time lost in four years is 23 h. 15 min. 18.8 sec.—that is, *one day*, lacking only 44 min. 41.2 sec.; hence, the first error can be corrected by adding *one* day every *four* years, making the year to consist of 366 days.

II. If every fourth year be reckoned as leap year, since we add 44 min., etc. too much, the time *gained* in the calendar in four years is 44 min. 41.2 sec., and in 100 years it will be 18 h. 37 min. 10 sec.—that is, *one* day, lacking 5 h. 22 min. 50 sec.; hence, the second error may be corrected by deducting one day from each centennial leap year, thus calling each centennial year a common year of 365 days.

III. Again, if every centennial year be reckoned as a common year, since we do not add enough, the time lost in 100 years will be 5 h. 22 min. 50 sec., and in 400 years it will be 21 h. 31 min. 20 sec.; hence, the time lost in 400 years will be *one* day lacking 2 h. 28 min. 40 sec., and this error may be rectified by making every fourth centennial year a leap year. In the same way we may make the calendar correct for any number of years.

NOTE.—The reckoning of time by the ancients was very inaccurate. The calendar was reformed by Julius Cæsar, 46 B. C., who made the year to consist of 365¼ days, adding one day every fourth year. This correction assumed the year to consist of 365 days 6 hours, which is 11 min. 10.3 sec. too much; hence his correction introduced a slight error, which in 1582 had amounted to about 14 days. In 1582, Pope Gregory corrected the error which arose from the above correction by striking out 10 days from the calendar, so as to bring the vernal equinox on March 21, as it was at the time of the Council of Nice, 325 A. D., calling the 5th of October the 15th, and ordaining that henceforth only those centennial years should be leap years which are divisible by 400. This change was soon adopted by most Catholic countries. Great Britain did not make the change till 1752, when by Act of Parliament the 3d of September was made the 14th, the error then amounting to 11 days. Russia and the other countries of the Greek Church still adhere to the Julian Calendar, their dates being about 12 days behind ours. The dates are distinguished as Old Style and New Style, marked O. S. and N. S. respectively. In the Old Style the *civil* or *legal* year commenced on the 25th of

March, while the *historical* year commenced on the 1st of January, and dates between those days were marked with the number of both years; as, Jan. 30, $164\frac{8}{9}$. The New Style made the civil year commence also on the 1st of January.

UNITED STATES MONEY.

I. Name.—United States money is so called because it is the money of the United States. It is called *Federal Money* because it was the money of the Federal Union. It was adopted by Act of Congress, Aug. 8, 1786.

II. Terms.—The term *dollar* is supposed to be from *Dale* or *Daleburg*, a town where it was first coined; *dime* is from the French *disme*, meaning *a tenth*; *cent* is from the Latin *centum*, *a hundred*; *mill* is from the Latin *mille*, *a thousand*; *eagle* is from the name of the national bird. The *cent* was proposed by Robert Morris and named by Thomas Jefferson.

IV. Coins.—The coins are of *gold*, *silver*, *nickel*, and *bronze*. The *gold coins* are the *double-eagle*, *eagle*, *half-eagle*, *quarter-eagle*, and *one dollar*. The *silver coins* are the *dollar*, *half-dollar*, *quarter-dollar*, and *dime*. The *nickel coin* is the *five-cent piece*. The *bronze coin* is the *cent*. The gold three-dollar piece, the silver twenty-cent piece, half-dime, and three-cent piece, the nickel three-cent piece and cent, the bronze two-cent piece, and the old copper cent and half-cent, although still occasionally seen in circulation, are no longer coined. The mill has never been a coin; it is merely a convenient name for the tenth part of a cent.

V. Composition.—The gold and silver coins consist of 9 parts of pure metal and 1 part alloy. The alloy of the silver coin consists of pure copper; the alloy of the gold coin consists of silver and copper, the silver not to exceed $\frac{1}{10}$ of the alloy. The nickel coins contain $\frac{1}{4}$ nickel and $\frac{3}{4}$ copper. The bronze coins consist of 95 parts copper and 5 parts tin and zinc.

VI. Gold coins are a legal tender for any amount; silver coins of the present coinage, for any amount not exceeding $5 in any one payment; bronze and nickel coins, for any amount not exceeding 25 cents in any one payment.

ENGLISH MONEY.

I. Name.—The term *Sterling* is supposed to be derived from *Easterling*, the name given to early German traders, who came from the East to England. Their money was called Easterling Money, which was contracted into *Sterling Money*.

II. Terms.—The term *farthing* is a modification of "four things," the old English penny being marked with a cross so deeply impressed that it could be broken into two or four pieces, called respectively *half-penny* and *four things*. The *pound* as a measure of value was derived from the pound as a measure of weight, 240 pence formerly weighing a pound. The *guinea* is so called because the gold of which it was first made came from Guinea, in Africa.

III. Symbols.—The symbols £, s., d., qr. are the initials of the Latin words *libra*, *solidus*, *denarius*, and *quadrans*, signifying respectively pound, *shilling*, penny, and quarter.

V. Coins.—The coins are of three classes—*gold*, *silver*, and *copper*. The *gold coins* are the *sovereign* (= £1) and *half-sovereign* (= 10 s.), *guinea* (= 21 s.) and *half-guinea* (=10 s. 6 d.). The *silver coins* are the *crown* (= 5 s.), the *half-crown* (= 2 s. 6 d.), the *florin* (= 2 s.), the *shilling*, and the *six-penny*, *four-penny*, and *three-penny* pieces. The *copper coins* are the *penny*, *half-penny*, and *farthing*. The guinea and half-guinea, crown and half-crown, though still seen in circulation, are no longer coined.

LUMBERMEN'S NOTATION.

657. Lumbermen in marking lumber employ a modification of the Roman Method of Notation. The first four characters are like the Roman. The others are as follows:

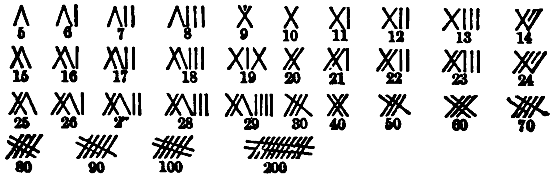


TABLE OF COMPOUND INTEREST.

Amount of $1 *at Compound Interest in any number of years not exceeding* 20.

Yr.	2½ per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.0250 0000	1.0300 0000	1.0400 0000	1.0500 000	1.0600 000	1.0700 000
2	1.0506 2500	1.0609 0000	1.0816 0000	1.1025 000	1.1236 000	1.1449 000
3	1.0768 9062	1.0927 2700	1.1248 6400	1.1576 250	1.1910 160	1.2250 430
4	1.1038 1289	1.1255 0881	1.1698 5856	1.2155 063	1.2624 770	1.3107 960
5	1.1314 0821	1.1592 7407	1.2166 5290	1.2762 816	1.3382 256	1.4025 517
6	1.1596 9342	1.1940 5230	1.2653 1902	1.3400 956	1.4185 191	1.5007 304
7	1.1886 8575	1.2298 7387	1.3159 3178	1.4071 004	1.5036 303	1.6057 815
8	1.2184 0290	1.2667 7008	1.3685 6905	1.4774 554	1.5938 481	1.7181 862
9	1.2488 6297	1.3047 7318	1.4233 1181	1.5513 282	1.6894 790	1.8384 592
10	1.2800 8454	1.3439 1638	1.4802 4428	1.6288 946	1.7908 477	1.9671 514
11	1.3120 8666	1.3842 3387	1.5394 5406	1.7103 394	1.8982 986	2.1048 520
12	1.3448 8882	1.4257 6089	1.6010 3222	1.7958 563	2.0121 965	2.2521 916
13	1.3785 1104	1.4685 3371	1.6650 7351	1.8856 491	2.1329 283	2.4098 450
14	1.4129 7382	1.5125 8972	1.7316 7645	1.9799 316	2.2609 040	2.5785 342
15	1.4482 9817	1.5579 6742	1.8009 4351	2.0789 282	2.3965 582	2.7590 315
16	1.4845 0562	1.6047 0644	1.8729 8125	2.1828 746	2.5403 517	2.9521 638
17	1.5216 1826	1.6528 4763	1.9479 0050	2.2920 183	2.6927 728	3.1588 152
18	1.5596 5872	1.7024 3306	2.0258 1652	2.4066 192	2.8543 392	3.3799 323
19	1.5986 5019	1.7535 0605	2.1068 4918	2.5269 502	3.0255 995	3.6165 275
20	1.6386 1644	1.8061 1123	2.1911 2314	2.6532 977	3.2071 355	3.8696 845

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